

# ASSESSING BEGINNING TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) IN PRACTICE

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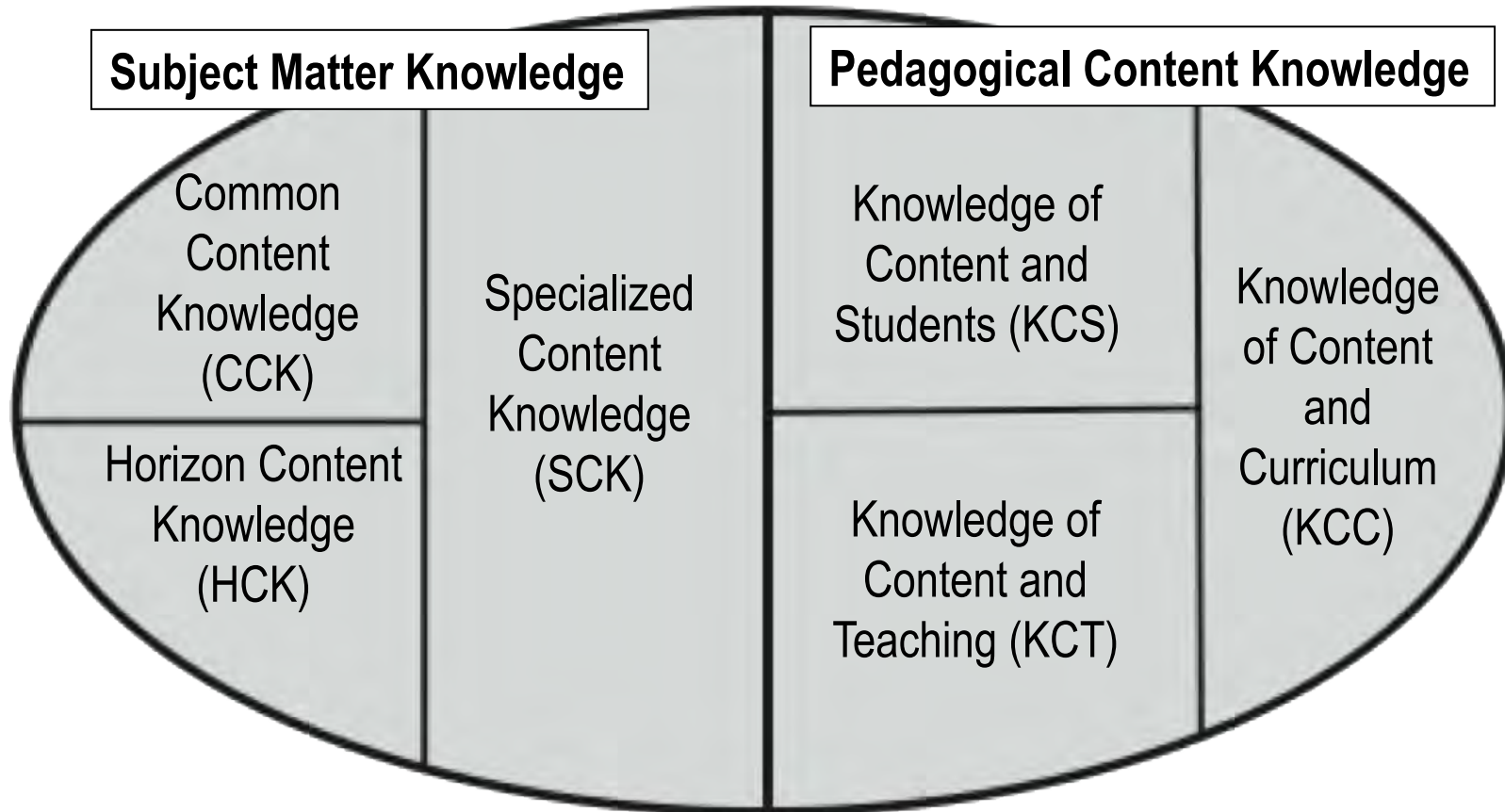
# THE CHALLENGE

- Beginning teachers often lack skills and knowledge needed for responsible entry-level teaching.
- Students, parents, and schools need beginning teachers who are ready for classroom practice.
- Policymakers and critics demand rigorous assessments of early career teachers' effectiveness.
- The profession needs to establish appropriate means for verifying beginning teachers' skills and knowledge.

# WHAT DO ASSESSMENTS OF BEGINNING TEACHERS' CAPABILITIES NEED TO BE LIKE?

- Assess entry-level practice: Actual skills and knowledge for doing teaching
- Provide information about beginning teachers' development and about instructional needs
- Be useful to beginner teachers and program instructors, and also demonstrate professional accountability and rigor to external stakeholders
- Use time efficiently and resources wisely

# ASSESSING MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)



# ASSESSING CONTENT KNOWLEDGE AND TEACHING PRACTICES

Must also assess MKT as it is actually used in the work of teaching, for example:

- Choosing examples
- Posing questions
- Interpreting students' thinking and choosing what to say or do in response
- Explaining ideas to students
- Using representations and materials to show the meaning of mathematical ideas to students
- Leading productive class discussions about mathematical ideas and solutions

# CONSIDERING EXAMPLE ASSESSMENTS

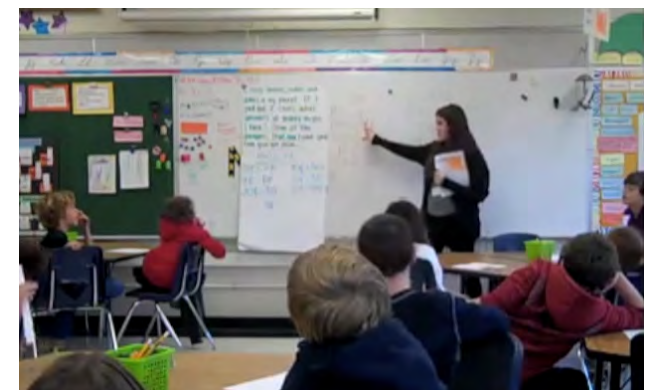
- ① Selected response assessment
- ② Simulation assessment
- ③ Field-embedded performance assessment

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ +100 \\ \hline +600 \\ \hline 875 \end{array}$

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3





# ATTENDING TO MKT THROUGH SELECTED RESPONSE ASSESSMENTS

*\*Special thanks to Merrie Blunk for her analysis of the data shared in this section.*



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# ASSESSMENT OVERVIEW

**Focus:** Mathematical knowledge for teaching for elementary grades Number Concepts and operations

- Common content knowledge (CCK)
- Specialized content knowledge (SCK)

**Timing:** As a pre-assessment at the beginning of the program; post-assessment at the end of the program



# WHAT ARE BEGINNING TEACHERS ASKED TO DO?

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

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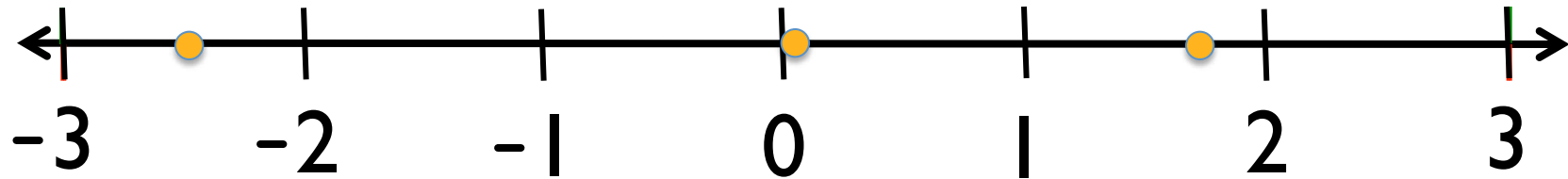
# WHICH OF THESE STUDENTS IS USING A METHOD THAT COULD BE USED TO MULTIPLY ANY TWO WHOLE NUMBERS?<sup>1</sup>

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600 \\ \hline 875 \end{array}$

<sup>1</sup>Measures copyright 2001, Learning Mathematics for Teaching/Study of Instructional Improvement (SII)/ Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of SII. Measures development supported by NSF grants REC-9979873, EHR-0233456 and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

# WHAT ARE WE LEARNING FROM THIS ASSESSMENT?

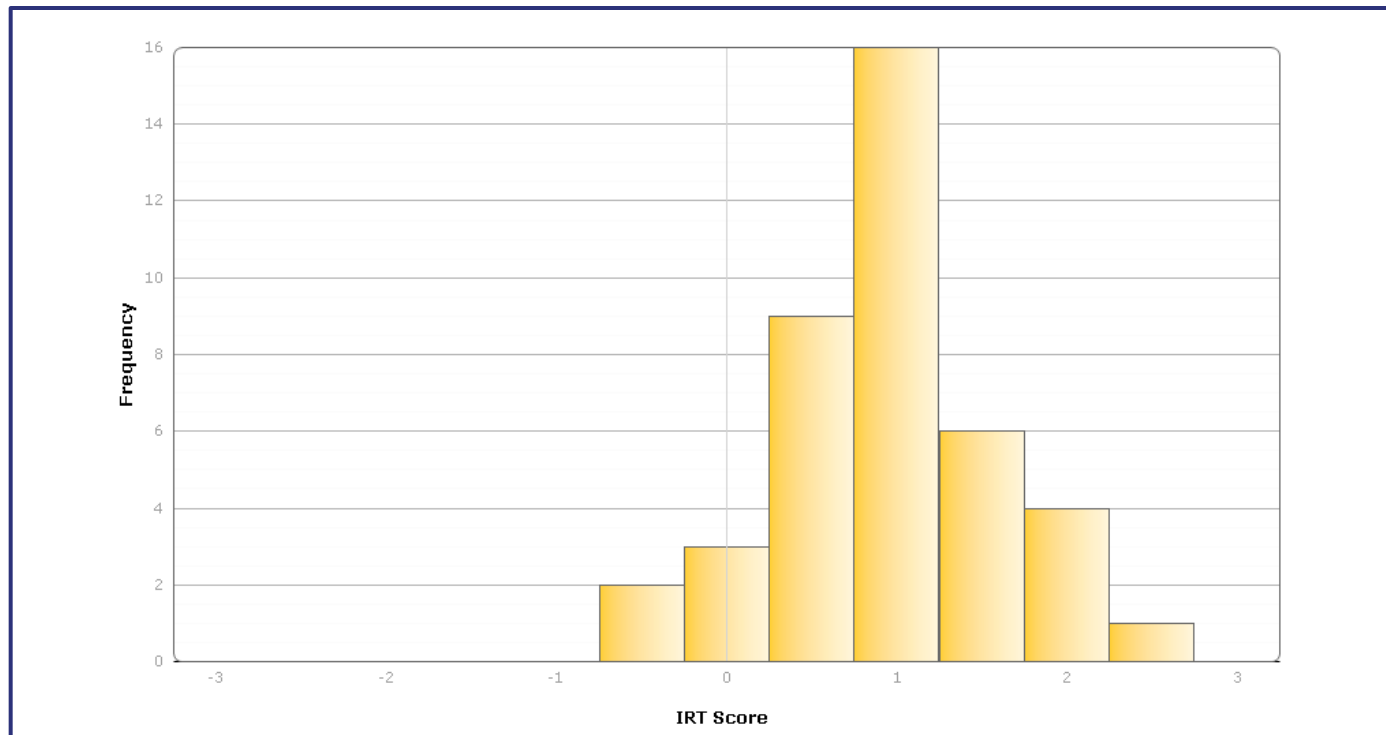
## *PRETEST*



N=104    mean = .06    min= -2.47    max= 1.84    sd= 0.77

# WHAT ARE WE LEARNING FROM THIS ASSESSMENT?

## POST TEST



n=41 min= -0.557 max= 2.513 mean= 0.969 sd= 0.643

# USING THIS ASSESSMENT

Using a selected response assessment allows us to:

- Assess the same MKT territory for all interns
- Understand the MKT of interns as a group, but not as reliable for understanding the MKT of particular interns
- Gather information for program level purposes

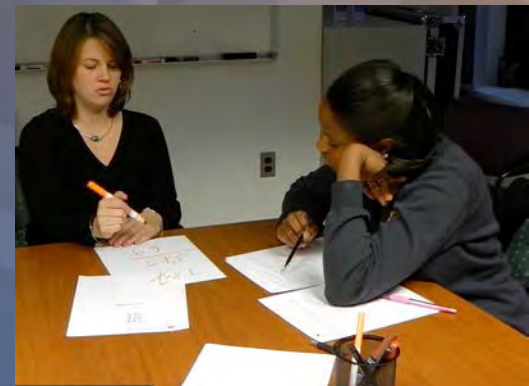
Using this assessment has revealed that our beginning teachers:

- Enter the program with MKT levels similar to the “average” U.S. teacher
- Gain about one standard deviation worth of MKT in the program
- Perform differently depending on the type of problems

# ATTENDING TO MKT THROUGH SIMULATION ASSESSMENT



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# ELICITING AND INTERPRETING STUDENT THINKING

**A core teaching practice:** To find out what students know or understand, and how they are thinking/reasoning

- Posing questions to get student to talk
- Listening to and hearing what student says
- Posing questions to probe
- Developing an idea of what student thinks
- Checking one's interpretation or pressing to learn more by extending or posing questions

# WHY TRY SIMULATIONS?

- **Standardization:** Appraises on-demand rather than at beginning teacher's discretion
- **Parity:** Makes possible fairness with respect to specific contextual aspects
- **Detail:** Enables specification of content, situation, teaching “problem” to ensure that important aspects of teaching are being assessed

# ASSESSING SKILLS OF ELICITING AND INTERPRETING STUDENT THINKING

## CONTEXT

- **Focus:** Eliciting and interpreting student thinking with particular mathematics content
  - Provides an opportunity to assess mathematical knowledge for teaching
- **Timing:** Mid-program; after coursework focused on eliciting and interpreting student thinking

## ASSESSMENT OVERVIEW

A beginning teacher:

- Interacts with a “standardized student” about a sample of student work
- Responds to a series of follow-up questions to surface the intern’s
  - interpretation of the student’s thinking
  - hypothesis about how the student would perform on a similar task

# OPPORTUNITIES TO SEE MKT IN USE

When we assess beginning teachers' skills with eliciting and interpreting a student's thinking, we can learn about their use of mathematical knowledge for teaching (MKT) as they —

- focus on specific elements of the mathematics in the problem
- word particular questions
- figure out an additional problem to probe the student's thinking
- predict how a student might approach a similar problem

# SETTING THE STAGE FOR ELICITING AND INTERPRETING

The beginning teacher:

1. prepares for an interaction with a standardized student about one piece of student work

Your goal is to elicit and probe to find out what the “student” did to produce the answer as well as the way in which the student understands the steps that were performed.

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

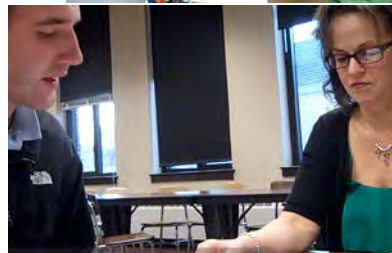
**Correct answer, alternative algorithm, degree of understanding is unclear**

# HOW IS EVIDENCE OF ELICITING SKILLS AND MKT OBTAINED?

$$\begin{array}{r} 78\overset{14}{4} \\ - 3\overset{2}{1}5 \\ \hline 469 \end{array}$$

The teaching intern:

1. prepares for an interaction with a standardized student about one piece of student work
- 2. interacts with the student to probe the standardized student's thinking**



## A Standardized Student

Developed response guidelines focused on:

- what the student is thinking such as
  - uses a method not conventional in the U.S. (but that is standard in many European and South American countries)
  - applies the method correctly and has conceptual understanding of the procedure
- general orientations towards responses such as
  - talk about digits in columns in terms of the place value of the column (e.g., 14 ones)
  - give the least amount of information that is still responsive to the question
- responses to anticipated questions



# ELICITING STUDENT THINKING: VIEWING FOCUS

What opportunities exist to assess the beginning teacher's mathematical knowledge for teaching and what do you notice about his MKT?

# ELICITING A STUDENT'S THINKING

$$\begin{array}{r} 784 \\ - 3215 \\ \hline 469 \end{array}$$

$$\begin{array}{r} 657 \\ - 276 \\ \hline 427 \end{array}$$



# ELICITING STUDENT THINKING: VIEWING FOCUS

$$\begin{array}{r} 78\cancel{4}^{14} \\ - 3\cancel{1}5 \\ \hline 469 \end{array}$$

What opportunities exist to assess the beginning teacher's mathematical knowledge for teaching and what do you notice about his MKT?

- Probes mathematics that is crucial for understanding the method
  - Does the student understand why adding 10 ones to the minuend and 1 ten to the subtrahend results in the same difference?
- Poses an additional task that is useful for confirming the student's method

$$\begin{array}{r} 65\cancel{7}^{13} \\ - 2\cancel{7}6 \\ \hline 427 \end{array}$$

# HOW IS EVIDENCE OF INTERPRETATION AND MKT OBTAINED?

The teaching intern:

1. prepares for an interaction with a standardized student about one piece of student work
2. interacts with the student to probes the standardized student's thinking
3. **responds to questions about her/his interpretation of the student's thinking, including predicting the student's response on a similar task**

## Questions with an MKT focus

- a) Posing another mathematics task:
  - Another problem was posed: why were the numbers were selected
  - Another problem was NOT posed: identify a problem that would be useful and whyApplying the student's method to a similar problem

$$\begin{array}{r} 761 \\ - 342 \\ \hline \end{array}$$

- b) Explain whether the method is generalizable and why

# INTERPRETING STUDENT THINKING: VIEWING FOCUS

What opportunities exist to assess the beginning teacher's mathematical knowledge for teaching and what do you notice about his MKT?

# INTERPRETING STUDENT THINKING

$$\begin{array}{r} 784 \\ - 3215 \\ \hline 469 \end{array}$$

$$\begin{array}{r} 653 \\ - 276 \\ \hline 427 \end{array}$$





# INTERPRETING STUDENT THINKING: VIEWING FOCUS

What opportunities exist to assess the beginning teacher's mathematical knowledge for teaching and what do you notice about his MKT?

- Mathematical justification for the additional task that was posed during the eliciting part
- Explanation for why the method always works

$$\begin{array}{r} 65\cancel{3}^{13} \\ - 276 \\ \hline 427 \end{array}$$

# USING THIS ASSESSMENT

Using a simulation of a teaching practice allows us to

- Assess MKT and pedagogical skill with eliciting and interpreting
- Provide feedback to individual beginners while also learning about the group as a whole
- Information to guide continued instruction and support

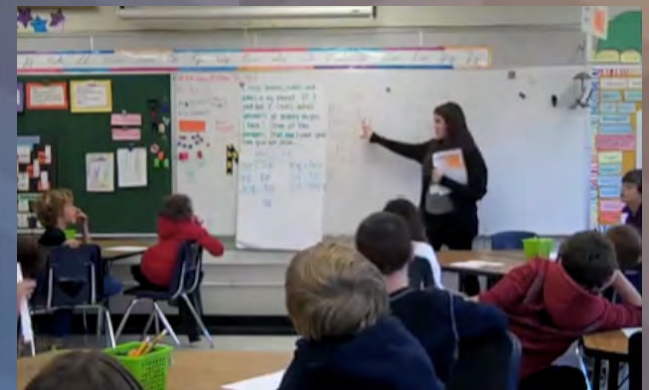
Learning from and about this assessment

- Reveals differences among beginning teachers with respect to generating mathematically similar problems
- Reaffirms that skill in eliciting does not necessarily translate into skill with interpreting
- This form of assessment provides substantial leverage on persistent challenges (time, resources, fairness, and judgment)

# ATTENDING TO MKT THROUGH FIELD-EMBEDDED PERFORMANCE ASSESSMENT



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# ASSESSMENT OF THE ABILITY TO GIVE AN EXPLANATION

## CONTEXT

- **Focus:** Giving an explanation of the solution to a mathematics problem, which occurs in a problem-based mathematics discussion
  - Provides an opportunity to assess mathematical knowledge for teaching
- **Timing:** Third semester of a four-semester program; during math methods course
- **Location:** Occurs in field placement classroom

## ASSESSMENT OVERVIEW

A teaching intern:

- Sets up a mathematics problem
- Has students work independently on the problem
- Leads a discussion of the problem, including providing an explanation

# FEATURES OF “GOOD” EXPLANATIONS

1. Has a clear **purpose**: What is being claimed and justified
2. Has a logical **structure**: One point follows from another
3. Uses **representations and language** clearly and carefully
4. Focuses on **meaning** and is **geared to the person** to whom one is explaining (i.e., what do they already know and understand)

# A FOURTH GRADE MATHEMATICS TASK

I have pennies, nickels, and dimes in my pocket. If I pull out 2 coins, what amounts of money might I have? Show all the amounts that you found and how you got them.

**How could a teacher explain to fourth grade students that ALL of the amounts have been found?**

# CONTEXT FOR THE VIDEO

- Fourth grade classroom
- Students worked on the problem independently
- Intern led a discussion of the solutions, including:
  - eliciting all of the solutions
  - explaining that pulling two coins in two different orders results in the same amount (e.g., NP and PN)
  - stating that she is now going to explain why the amounts that the class has identified are the only amounts possible



# EXPLAINING MATHEMATICS: FOCUS QUESTION

What opportunities exist to assess the intern's mathematical knowledge for teaching and what do you notice about her MKT?

# EXPLAINING MATHEMATICS

$$PP = 2\text{¢}$$

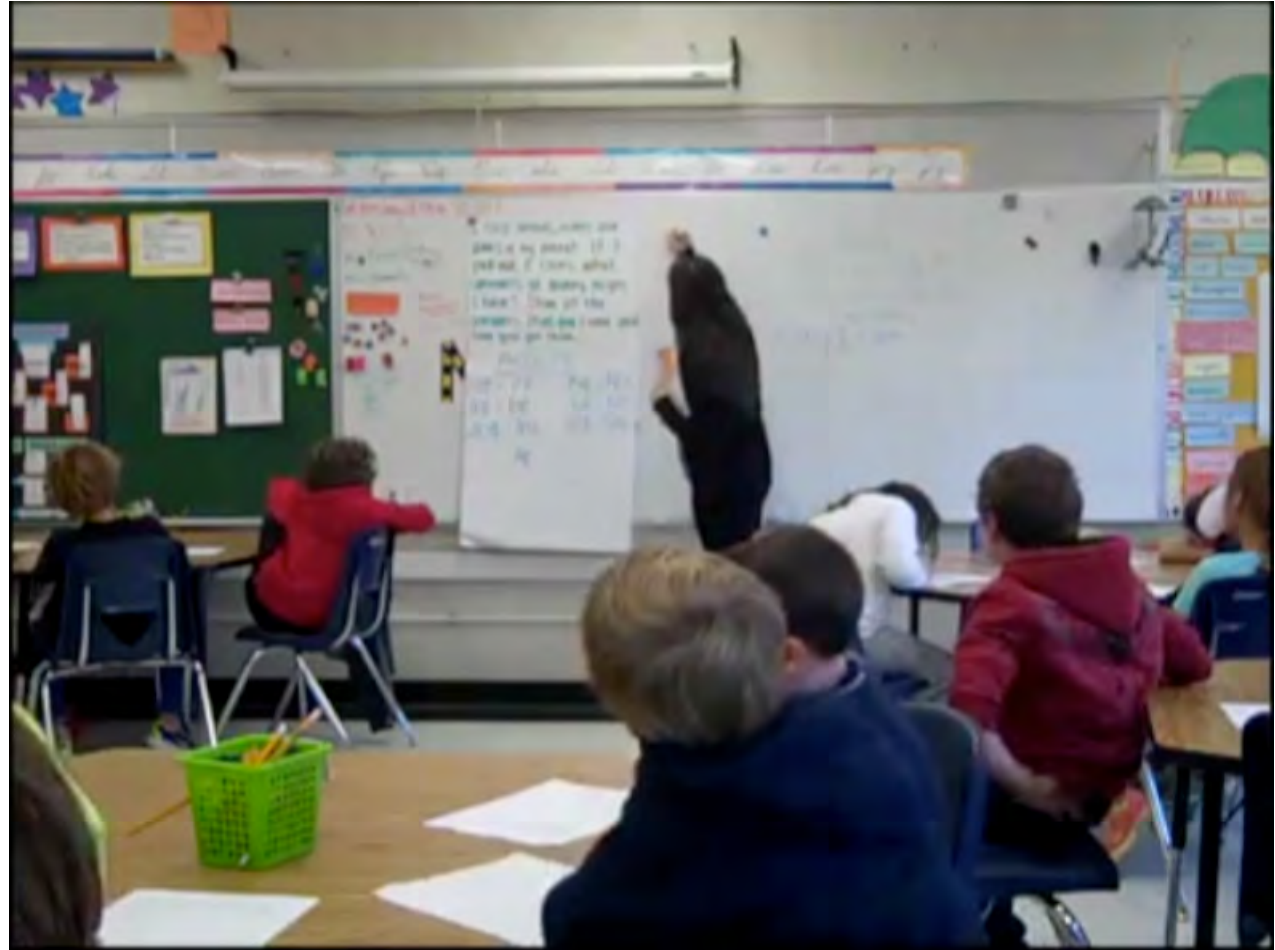
$$PD = 11\text{¢}$$

$$PN = 6\text{¢}$$

$$NN = 10\text{¢}$$

$$ND = 15\text{¢}$$

$$DD = 20\text{¢}$$



# EXPLAINING MATHEMATICS: VIEWING FOCUS

What opportunities exist to assess the intern's mathematical knowledge for teaching and what do you notice about her MKT?

- Shows all of the possible amounts
- Makes use of the features of good explanations:
  - Has a clear **purpose**
  - Has a logical **structure**
  - Uses **representations and language** clearly and carefully
  - Focuses on **meaning** and is **geared to the person** to whom one is explaining
- Articulates that 3 coins x 2 coins pulled = 6 amounts in this case, but 4 coins x 2 coins pulled does not result in only 8 amounts
  - Caution: She misinterpreted the student's statement

# USING THIS ASSESSMENT

Using a field-embedded assessment of teaching practice allows us to

- Assess MKT in use with actual students and a constrained set of tasks
- See each intern handling unique situations in classrooms
- Provide feedback and recommend particular resources that may be of use to further develop MKT

Learning from and about this assessment

- Many beginning teachers are able to embed explanations in their discussions
- Beginners are able to differentially adjust their explanations based on what students in the classroom say
- It is challenging to gain a coherent or systematic sense of the MKT of the group of beginners from this assessment

# NEXT STEPS



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# NEXT STEPS

- ① Establish justifiable thresholds for the proficiency of beginners at different points in their development
  - What is “fair” to expect of beginners on the simulation assessment after their first year in the program? Their second year?
- ② Analyze relationships between estimates of MKT and skill with particular teaching practices
  - Do we notice that beginners with high MKT typically elicit student thinking more completely or more insightfully interpret the elicited information?
- ③ Analyze the relationships among the estimates of MKT provided by different assessments



# QUESTIONS?



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