

ELICITING STUDENT THINKING: DECOMPOSING INTERACTIONAL INSTRUCTIONAL PRACTICES

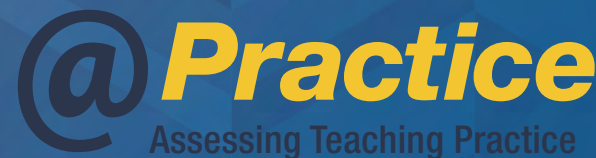
Sarah Kate Selling, Meghan Shaughnessy & Timothy Boerst

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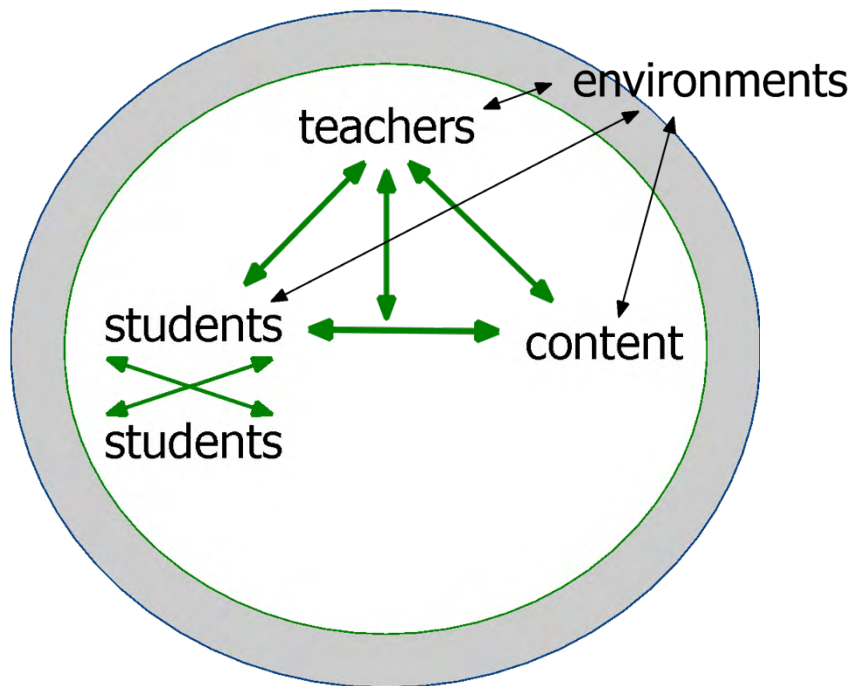
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THE GOAL: WELL-STARTED BEGINNING TEACHERS

Beginning teachers need:

- Deep and usable knowledge of academic content
- Skill with teaching practices routinely needed in daily work
- Knowledge of students as learners and as people
- Actionable professional commitment to the learning of all students
-

ENGAGING IN TEACHING PRACTICES



Teaching practices enable teachers to support students to engage with content in ways that are effective in, and sensitive to, a variety of educational environments

(Cohen, Raudenbush, and Ball, 2003; Lampert, 2001)

MAKING TEACHING PRACTICES LEARNABLE

Pedagogies of practice support professional learning using ways of:

- Representing practice that make it visible
- Breaking complex practices into their constituent parts (decomposition)
- Engaging in the practice in ways that approach full engagement (approximation)

(Grossman et al., 2009)

ZOOMING IN ON ONE PRACTICE: ELICITING STUDENT THINKING

To find out what students know or understand, and how they are thinking/reasoning, a teacher must:

- Establish an environment in which a student is comfortable sharing his/her thinking
- Pose questions to get students to talk
- Listen to and hearing what students say
- Probe students' responses
- Develop ideas about what a student thinks
- Check one's interpretation

LEARNING ABOUT PRESERVICE TEACHERS' ELICITING OF STUDENT THINKING

What does it look like and sound like when preservice teachers are eliciting student thinking:

- In a particular subject area – mathematics?
- In relation to particularly key ideas – algorithms?
- At particular points in the development of the idea – becoming proficient with an algorithm?

Do particular contexts influence eliciting? If so, how?

- Similarities and differences across contexts

USING SIMULATIONS

Simulations are approximations of practice that can be used for both assessing and supporting ongoing learning.

Simulations:

- Place authentic, practice-based demands on a participant
- Purposefully suspend or standardize some elements of the practice-based situation
- Are commonly used in many professional fields
- Can provide insights that are not possible or practical to determine in real-life professional contexts

OVERVIEW

- ① Exploring a simulation of eliciting a student's thinking
- ② Learning about preservice teachers' eliciting of a student's thinking

Scenario 1: A student's approach to the standard addition algorithm

Scenario 2: A student's use of an alternative subtraction algorithm

- ③ Examining commonalities and differences in eliciting across scenarios
- ④ Considering implications for teacher education

SCENARIO 1: A STANDARD ADDITION ALGORITHM



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EXPLORING THE TASK

- Examine the third grade student's work
- Anticipate what the student was likely thinking
- Generate questions to ask the student to:
 - Elicit *what* the “student” did to produce the answer
 - Probe what the student *understands* about the process used and the mathematical ideas underlying that process.

$$\begin{array}{r} 147 \\ + 328 \\ + 269 \\ \hline 734 \end{array}$$

SETTING THE STAGE FOR ELICITING

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

Your goal is to elicit and probe to find out what the “student” did to produce the

$$\begin{array}{r} \\ 1 \ 4 \ 7 \\ 3 \ 2 \ 8 \\ + 2 \ 6 \ 9 \\ \hline 7 \ 3 \ 4 \end{array} \Big)_{17}$$

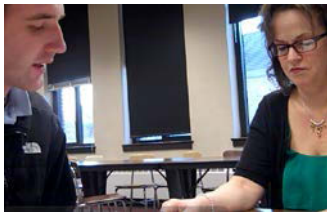
answer as well as the way in which the student understands the steps that were performed.

Incorrect answer, standard algorithm, degree of understanding is unclear

HOW IS EVIDENCE OF ELICITING SKILLS OBTAINED?

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work
2. **Interacts with the student to probes the standardized student's thinking**



A Standardized Student

Developed response guidelines focused on:

- What the student is thinking such as
 - Uses a standard algorithm for addition except that the student does not regroup correctly when the sum in a given-place value is greater than 19
 - The student does not understand the connection between the number “carried” and then “tens value” of the sum of the digits in the column to the right
- General orientations towards responses
- Responses to anticipated questions

$$\begin{array}{r} \\ 1 \ 4 \ 7 \\ + 2 \ 6 \ 9 \\ \hline 7 \ 3 \ 4 \end{array} \Big)_{17}$$

ELICITING STUDENT THINKING

What can we notice about this preservice teacher's skill with eliciting student thinking?

Evaluate whether the preservice teacher:

- Launches the interactions with a question that is neutral, open, and focused on student thinking
- Elicits the specific steps of the student's process
- Elicits the student's understanding of the steps
- Attends to the students' ideas in follow-up questions
- Uses appropriate tone and manner

$$\begin{array}{r} 147 \\ 328 \\ + 269 \\ \hline 734 \end{array} \quad \left. \vphantom{\begin{array}{r} 147 \\ 328 \\ + 269 \\ \hline 734 \end{array}} \right) 17$$

ELICITING A STUDENT'S THINKING

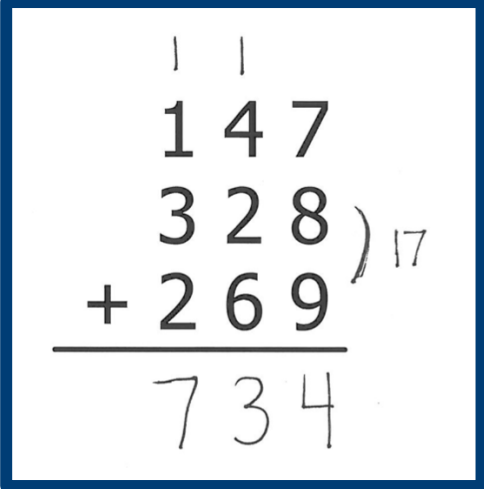


What is the teacher doing to elicit this student's thinking?

POSSIBLE OBSERVATIONS

The teacher candidate:

- Launched the interactions with a question that is neutral, open, and focused on student thinking
- Elicited specific steps of the student's process but did not ask about the order in which the student added digits within a column
- Probed the student's understanding of meaning of digits, why carrying, what "carry" means
- Attends to the students' ideas in follow-up questions
- Tone and manner: Uses evaluative language



A handwritten addition problem on a white background with a blue border. The problem is:

$$\begin{array}{r} 147 \\ +328 \\ \hline 734 \end{array}$$

The digits are written in black ink. There are vertical lines above the 1 and 3. A horizontal line is drawn under the 269. To the right of the 8, there is a closing parenthesis and the number 17, indicating a carry of 17 to the next column.

INITIAL SKILL IN ELICITING STUDENT THINKING

Context:

- End of program simulation assessment (23 interns)

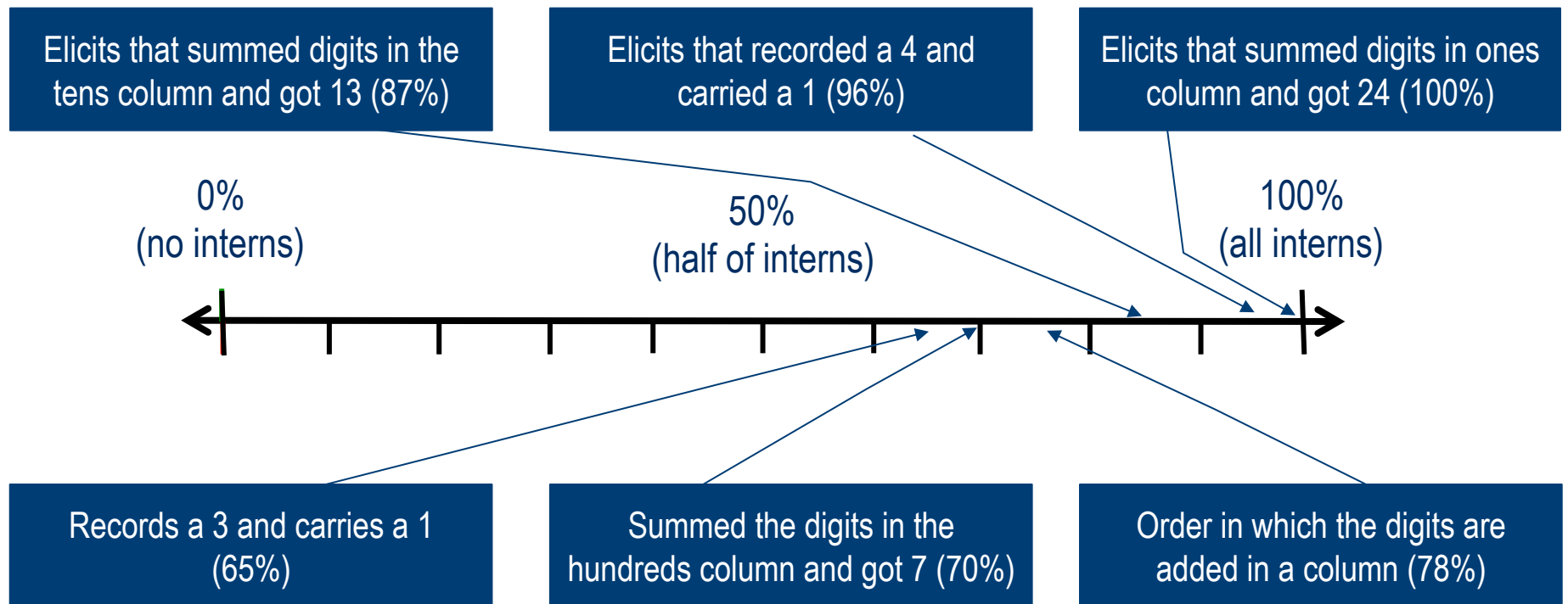
Analyzing the prevalence of eliciting moves:

- Eliciting components of the student's process
- Probing the student's understanding of the process
- Encouraging and attending to what the student says and writes
- Posing a purposeful follow-up problem



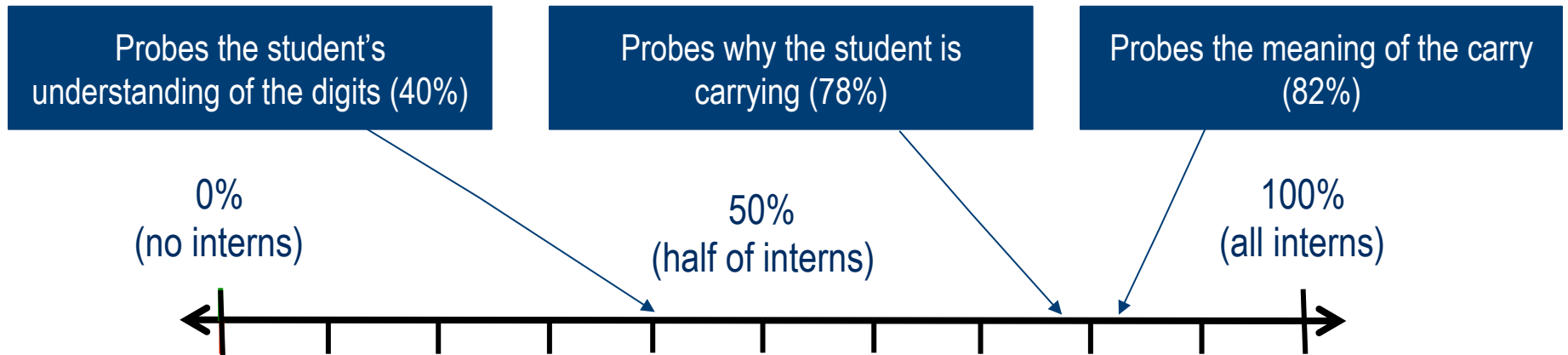
PREVALENCE OF MOVES TO ELICIT THE STUDENT'S PROCESS

$$\begin{array}{r} 11 \\ 147 \\ 328 \\ + 269 \\ \hline 734 \end{array}$$



PREVALENCE OF MOVES TO PROBE THE STUDENT'S UNDERSTANDING

$$\begin{array}{r}
 11 \\
 147 \\
 328 \\
 + 269 \\
 \hline
 734
 \end{array}$$



96% probed understanding of at least one component

78% probed understanding of two or more components

26% probed understanding of all three components

SCENARIO 2: ALTERNATIVE SUBTRACTION ALGORITHM



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EXPLORING THE TASK

- Examine the fourth grade student's work
- Anticipate what the student was likely thinking
- Generate questions to ask the student to:
 - Elicit *what* the “student” did to produce the answer
 - Probe what the student *understands* about the process used and the mathematical ideas underlying that process.

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

SETTING THE STAGE FOR ELICITING AND INTERPRETING

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

Add 10 ones

$$\begin{array}{r} 784 \\ - 315 \\ \hline \end{array}$$

Your goal is to elicit and probe to find out what the “student” did to

produce the answer as well as the way in which the student understands the steps that were performed.

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

Correct answer, alternative algorithm, degree of understanding is unclear

SETTING THE STAGE FOR ELICITING AND INTERPRETING

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

How can the difference between the two numbers be re-established?

$$\begin{array}{r} 784 \\ - 315 \\ \hline \end{array}$$

Your goal is to elicit and probe to find out what the “student” did to

produce the answer as well as the way in which the student understands the steps that were performed.

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

Correct answer, alternative algorithm, degree of understanding is unclear

SETTING THE STAGE FOR ELICITING AND INTERPRETING

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

Your goal is to elicit and probe to find out what the “student” did to

produce the answer as well as the way in which the student understands the steps that were performed.

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

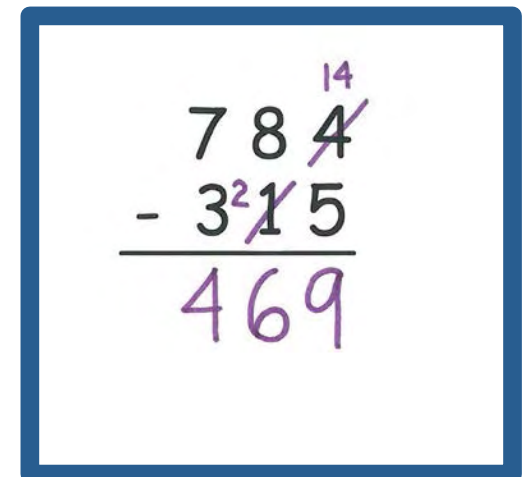
Correct answer, alternative algorithm, degree of understanding is unclear

ELICITING STUDENT THINKING

What can we notice about this preservice teacher's skill with eliciting student thinking?

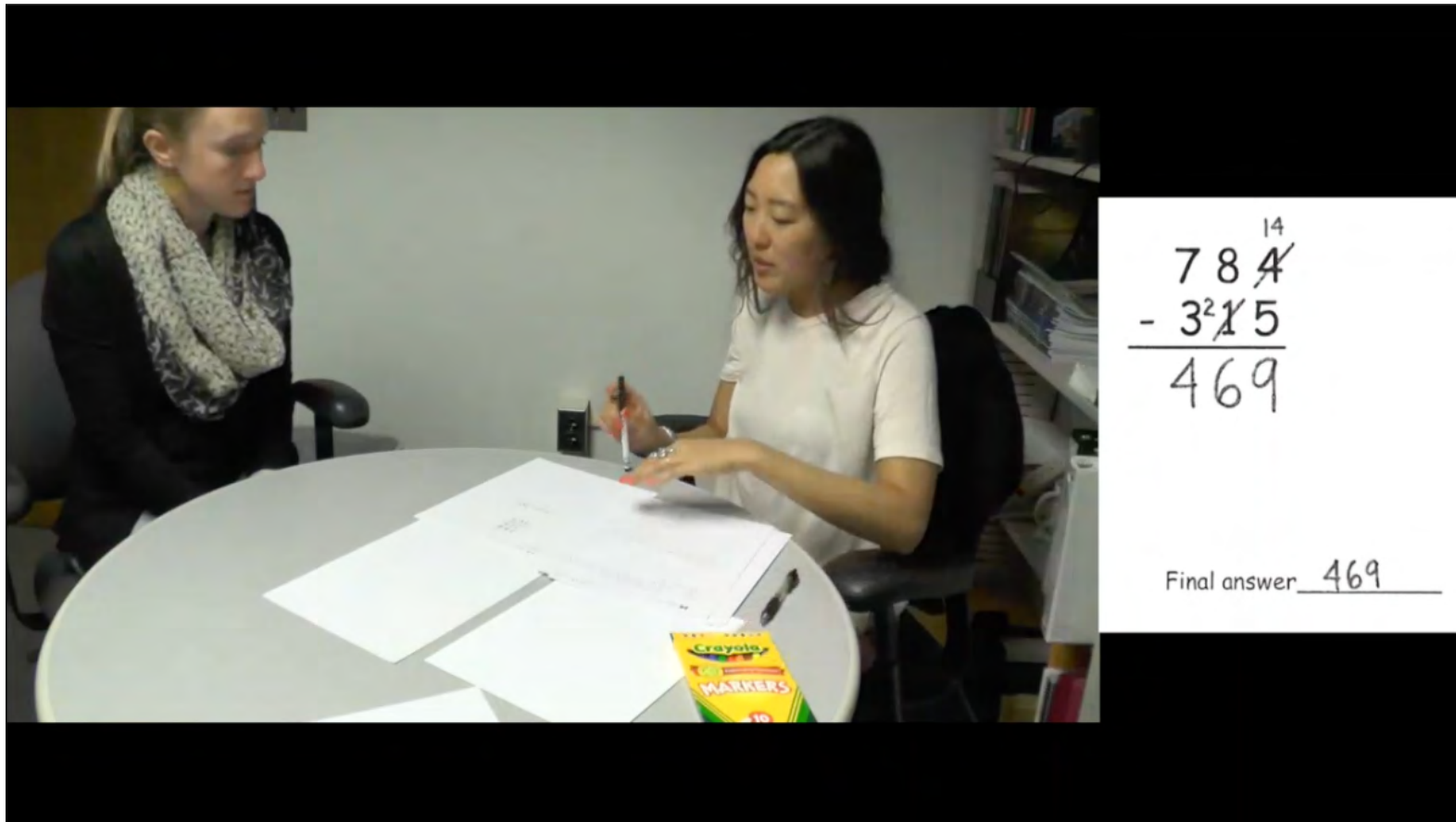
Evaluate whether the preservice teacher:

- Launches the interactions with a question that is neutral, open, and focused on student thinking
- Elicits the specific steps of the student's process
- Elicits the student's understanding of the steps
- Attends to the students' ideas in follow-up questions
- Uses appropriate tone and manner



A handwritten subtraction problem on a whiteboard. The problem is $784 - 315 = 469$. The numbers are written in purple. There are annotations: a purple '14' is written above the '4' in the top number, and a purple '2' is written above the '1' in the bottom number. The '4' in the top number and the '1' in the bottom number are crossed out with a purple slash. A horizontal line is drawn under the bottom number.

ELICITING A STUDENT'S THINKING



What is the teacher doing to elicit this student's thinking?

POSSIBLE OBSERVATIONS

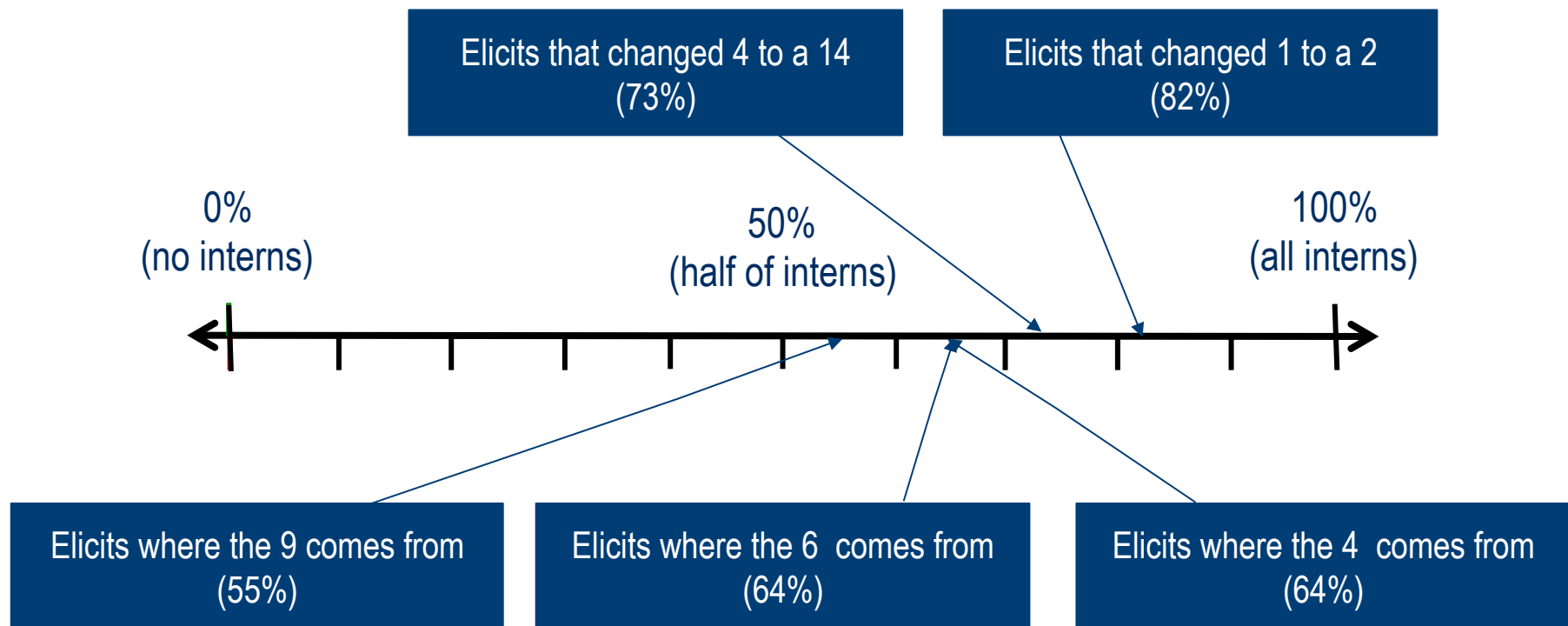
The teacher candidate:

- Launched the interactions with a question that is neutral, open, and focused on student thinking
- Elicited specific steps of the student's process but did not elicit all of the steps in completing the subtraction
- Probed around the 14 and 2, but did not fully probe how the student thought about changing both numbers
- Attends to the students' ideas in follow-up questions
- Tone and manner: Refrained from using evaluative language

$$\begin{array}{r} 78\cancel{4} \\ - 3\cancel{2}5 \\ \hline 469 \end{array}$$

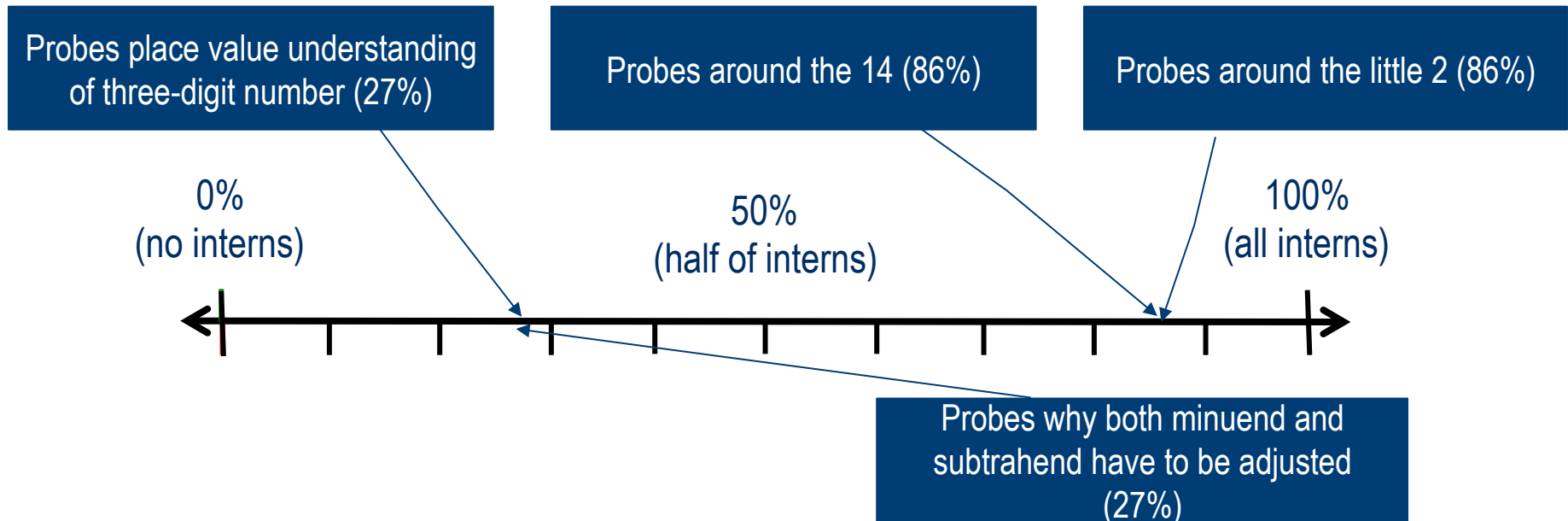
PREVALENCE OF MOVES TO ELICIT THE STUDENT'S PROCESS

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$



PREVALENCE OF MOVES TO PROBE THE STUDENT'S UNDERSTANDING

$$\begin{array}{r} 78\cancel{4}^{14} \\ - 3\cancel{2}^1 5 \\ \hline 469 \end{array}$$



100% probed understanding of at least one component

90% probed understanding of two or more components

15% probed understanding of all four components

EXAMINING COMMONALITIES AND DIFFERENCES IN ELICITING ACROSS SCENARIOS



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CROSS CASE ANALYSIS

- Compared individual teachers' eliciting across the two scenarios in matched pairs
- $N = 18$ pairs of videos



$$\begin{array}{r} 11 \\ 147 \\ 328 \\ + 269 \\ \hline 734 \end{array} \quad)_{17}$$



$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array} \quad \begin{array}{l} 14 \\ 2 \end{array}$$

MORE SIMILARITIES ACROSS SCENARIOS



$$\begin{array}{r} 11 \\ 147 \\ 328 \\ + 269 \\ \hline 734 \end{array} \Big)_{17}$$



$$\begin{array}{r} 78\cancel{4} \\ - 3\cancel{2}5 \\ \hline 469 \end{array} \begin{array}{l} 14 \\ \end{array}$$

- Posed a follow up problem
- Probed place value: “Can you read that number for me?”
- Asked whether the answer made sense and how to check
- Used similar discourse pattern in questioning

Addition standard algorithm

“Can I carry a 2 or a 3?”

“Let’s just say I added all of these and got 33...”

European subtraction

“So what if I...can I add ten ones to this number?”

“Would I be able to cross out the 3 for the ten ones?”

POSSIBLE QUALITATIVE DIFFERENCES

- How long did the teachers spend eliciting on each?
- How many questions were they asking?
- What were the teachers' questions focused on?

$$\begin{array}{r} 147 \\ + 328 \\ \hline 734 \end{array} \quad \left. \vphantom{\begin{array}{r} 147 \\ 328 \\ \hline 734 \end{array}} \right) 17$$

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$



$$\begin{array}{r}
 11 \\
 147 \\
 328 \\
 + 269 \\
 \hline
 734
 \end{array}
 \begin{array}{l}
) \\
 17
 \end{array}$$

3:00



$$\begin{array}{r}
 784 \\
 - 3215 \\
 \hline
 469
 \end{array}
 \begin{array}{l}
 14 \\
 \cancel{4}
 \end{array}$$

4:06

Can you first tell me where you began adding?
 Okay, these. What should we call these? Well, what place value?
 Can I just call them ones place value?
 We'll do ones, tens, hundreds. Is that okay?
 And I see you wrote seventeen. Did you get that by adding eight plus nine?
 Very nice. But where did you get the 4 from?
 Why do you have to carry a 1?
 Okay. So you would carry a one. 29 Can I carry a two or a three?
 Always a one. Okay. And then what did you do for the tens column?
 And then you wrote- Where is the three from?
 And you can't write the one here, so-
 What if I got- Let's just say I added all these and I got thirty-three. Is that how I do it?
 Okay. And now the hundreds column. Can you take me through that?
 And there's no two digits, so you can just write seven.
 Good. Can you read that number for me?
 And how- If I were to check my answer, what can I do? I want to make sure this is correct.
 Can I give you another problem?
 Can you walk me through it?
 Can you read me that number?
 And you- To check, you would just probably add it again, but you know your math is correct.
 Okay. All right, that's it.

So I was looking at your work 1 and this is what you did. And I noticed that you started by crossing out the four and replacing it with fourteen.
 Can you tell me where you got the fourteen from?
 And where did you get the ten ones from?
 What number?
 So, you changed three hundred fifteen to what?
 And why is that?
 Okay. So, what if I added- Can I add ten ones to this number?
 And then would the ten ones become the two or would I switch over?
 So if I- Let's scratch this out. I'm going to rewrite it for you.
 So, if I took ten ones and added it to the five, I would get?
 Fifteen. And where would I- Would I cross this one out and make it into a two or what would I do?
 Yeah, we're only adding ten ones. So, instead of adding-
 So, you knew to add ten ones here to the four?
 Why is that?
 Yeah, so it's- it would be like a negative number. So you knew- And then from here, you went on to change the one to a two to switch over the ten ones, would you say? And then- Can I- Would I be able to cross out the three instead of the one for the ten ones? What do you think?
 Why? How so?
 Okay. Can I ask you if I cross out the eight and made it into eighteen, then do I cross out the three and make it into a two? How would that work? If-
 Let's say I was doing this subtraction problem. And we're just looking at this.
 So, can I subtract nine from eight?
 So, what would I do in this situation?
 Okay. And then what would I do?
 A four. Okay. And why did you not cross out anything in the hundreds place value or I would say hundreds column?
 Uh-huh. So you knew you didn't need to do anything. And how can you check to make sure that your answer is correct? What would you do?
 So you're rounding? Is there another way that you can do it to get exactly four hundred and sixty-nine and know that that's the correct answer without rounding?
 Okay. Okay, that's it.



$$\begin{array}{r}
 11 \\
 147 \\
 328 \\
 + 269 \\
 \hline
 734
 \end{array}
 \begin{array}{l}
) \\
 17
 \end{array}$$

3:00



$$\begin{array}{r}
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 Okay. So you would carry a one. Can I carry a two or a three?
 Always a one. Okay. And then what did you do for the tens column?
 And then you wrote- Where is the three from?
 And you can't write the one here, so-
 What if I got- Let's just say I added all these and I got thirty-three. Is that how I do it?
 Okay. And now the hundreds column. Can you take me through that?
 And there's no two digits, so you can just write seven.
 Good. Can you read that number for me?
 And how- If I were to check my answer, what can I do? I want to make sure this is correct.
 Can I give you another problem?
 Can you walk me through it?
 Can you read me that number?
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 Okay. All right, that's it.

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 Can you tell me where you got the fourteen from?
 And where did you get the ten ones from?
 What number?
 So, you changed three hundred fifteen to what?
 And why is that?
 Okay. So, what if I added- Can I add ten ones to this number?
 And then would the ten ones become the two or would I switch over?
 So if I- Let's scratch this out. I'm going to rewrite it for you.
 So, if I took ten ones and added it to the five, I would get?
 Fifteen. And where would I- Would I cross this one out and make it into a two or what would I do?
 Yeah, we're only adding ten ones. So, instead of adding-
 So, you knew to add ten ones here to the four?
 Why is that?
 Yeah, so it's- it would be like a negative number. So you knew- And then from here, you went on to change the one to a two to switch over the ten ones, would you say? And then- Can I- Would I be able to cross out the three instead of the one for the ten ones? What do you think?
 Why? How so?
 Okay. Can I ask you if I cross out the eight and made it into eighteen, then do I cross out the three and make it into a two? How would that work? If-
 Let's say I was doing this subtraction problem. And we're just looking at this.
 So, can I subtract nine from eight?
 So, what would I do in this situation?
 Okay. And then what would I do?
 A four. Okay. And why did you not cross out anything in the hundreds place value or I would say hundreds column?
 Uh-huh. So you knew you didn't need to do anything. And how can you check to make sure that your answer is correct? What would you do?
 So you're rounding? Is there another way that you can do it to get exactly four hundred and sixty-nine and know that that's the correct answer without rounding?
 Okay. Okay, that's it.

KEY QUALITATIVE DIFFERENCES

- Time

- 4 spent similar times
- Of the remaining 14,

$$\begin{array}{r} 1 1 \\ 147 \\ + 328 \\ \hline 734 \end{array} \Big) 17$$

$$\begin{array}{r} 14 \\ 784 \\ - 3215 \\ \hline 469 \end{array}$$

- 10/14 spent markedly longer on European Subtraction
- 4/14 spent markedly longer on Addition Standard Algorithm

- Average times: 2:27 (addition) vs. 3:01 (subtraction)

- Dwelling on key mathematical parts

- 13/18 asked noticeably more questions about the key parts of European Subtraction

④ CONSIDERING IMPLICATIONS FOR TEACHER EDUCATION



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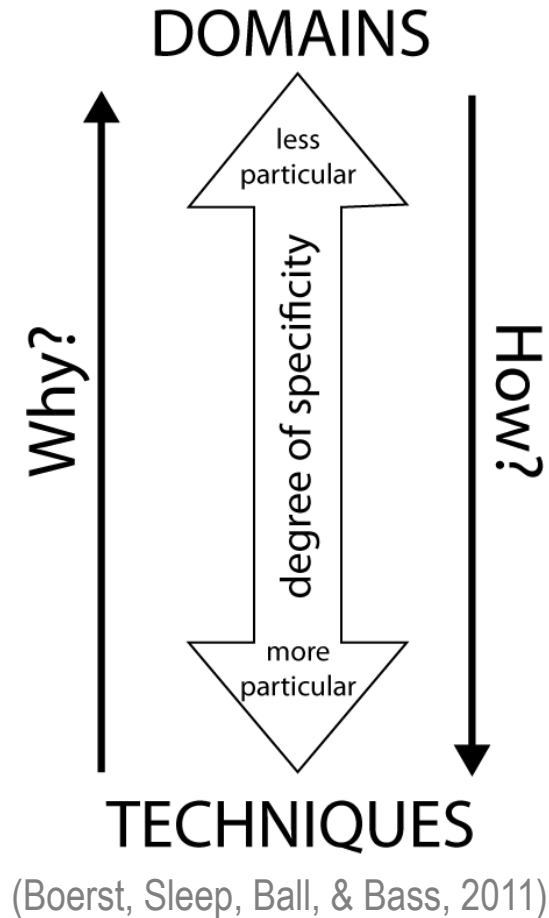
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REVISITING DECOMPOSITIONS OF PRACTICE

What degree of decomposition is appropriate for:

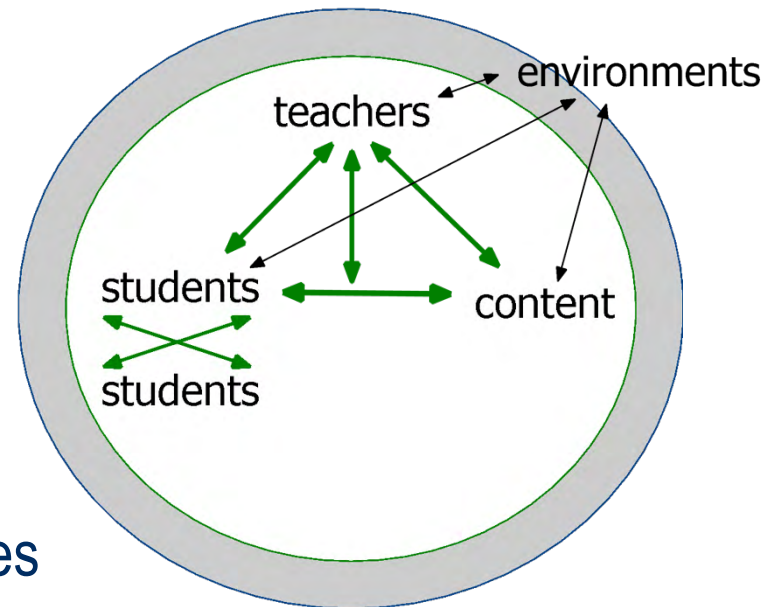
- Opportunities to learn about a teaching practice
- Evaluation of skill with a teaching practice
- Conversations among peers
- Reflection on practice



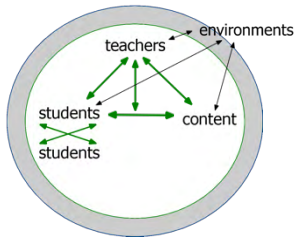
REVISITING THE INFLUENCE OF ENVIRONMENT ON TEACHING PRACTICE

Across what contexts would one want a teacher's eliciting to be enacted in similar ways:

- Different strategies
- Correct or incorrect answers
- Connection with the teacher's own preferences and knowledge
- Different curriculum
- Different content domains
- Different communities and/or cultures

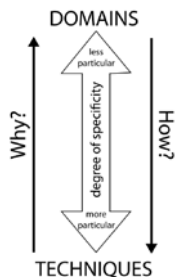


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