PRESERVICE TEACHERS' GENERALIZATIONS ABOUT AN AREA STRATEGY

@Practice
Assessing Teaching Practice

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BACKGROUND

- **Generalization:** Skillful mathematics teaching involves being able to size up children's strategies and determine whether a strategy would work in general (Ball, Thames, & Phelps, 2008). This may be particularly challenging for novice teachers who have less familiarity with nonstandard approaches.
- **Decomposition of Practice**: Teaching can be broken down into smaller parts that can be taught, studied, and rehearsed by preservice teachers. The parts must maintain their integrity so that they can be reintegrated into the practice of teaching (Grossman & Shahan, 2005).
- **Simulations:** Situations that represent a context of practice with fidelity and elicit authentic professional work. Used in the preparation of professionals in other fields. Focus on the doing of teaching while standardizing important contextual factors that impact both teaching and ability to appraise its quality.

RESEARCH FOCUS

- Generalizations about nonstandard approaches can reveal the nature of preservice teachers' understanding of specific mathematical ideas, as well as their ability to identify the core mathematics of a given problem type and strategy.
- Valuing and leveraging student thinking is imperative for teachers to teach mathematics equitably.
- What do preservice teachers attend to when generalizing about the validity of a nonstandard strategy for finding the area of a rectangle?

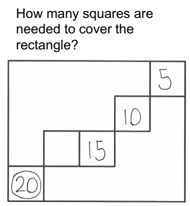
METHODS

- Participants: 38 preservice elementary teachers at three stages in a two-year teacher education program: pre-admission, beginning of Year 1, and beginning of Year 2.
- Simulation assessment.
- Analyzed preservice teachers' generalizations focusing on:
  - the problem features needed for the approach to work;
  - the scenarios in which the approach would *not* work; and
  - explanations about why the approach would not work in such scenarios.

SIMULATION ASSESSMENT

The simulation assessment consists of three parts:

- **Preparation:** Preparing for an interaction with a standardized student about a specific piece of student work.



- **Simulation:** Eliciting and probing the standardized student's thinking to understand the steps the student took, why the student performed the steps, and the student's understanding of the key mathematical ideas.
- **Interview:** Interpreting the student's thinking and using evidence from the simulation. This includes generalizing about the validity of the student's process:
  - What has to be true about a shape for this strategy to work?
  - What is a shape for which this strategy would not work? Why?

Student Role Protocol to Standardize the Assessment

The student's process:

- Uses marked squares to find the number of squares in one row and then skip counts by that number (Battista et al., 1998).
- **Core idea:** This process works for rectangles that have the same number of unit squares in each row.

The student understands:

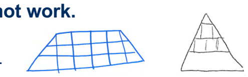
- Composite units (a set of smaller units that is treated as a single entity) and how they can be used to determine area iterating unit squares.
- Not all of the individual unit squares need to be marked to determine the number of squares needed to cover the rectangle.
- Covering the rectangle means placing unit squares without gaps or overlaps.

The student does NOT understand:

- The general meaning of area as the space inside a shape.
- Side lengths of a rectangle, or how side lengths can be used to calculate area.
- How to account for non-unit sized parts in the rectangle (i.e., fractional parts).
- How to decompose shapes into rectangles to find the area.

FINDINGS

- **Most preservice teachers identified relevant problem features needed for the student's process to work.**
  - 79% of preservice teachers clearly articulated the **core idea** that each row of the rectangle needs to contain the same number of unit squares.
  - The remaining 21% of preservice teachers discussed features that were not directly relevant to the core idea (e.g., having whole unit squares and no partial squares).
- **All participants accurately identified a shape for which the student's strategy would not work.**
  - Examples: Triangles, circles, trapezoids, composite shapes.



- **Explanations for why the student's strategy would not work for a given shape varied.**
  - 54% of preservice teachers explained that the strategy would not work for a given shape by addressing the **core idea** — the shape lacks the same number of unit squares in each row.
  - Other explanations focused on the potential difficulty of visualizing unit squares or the presence of partial squares.
  - Attention to the properties of shapes and use of precise mathematical language varied widely (e.g., saying "bent edges" or "small corners" in reference to non-right or acute angles).
  - Participants who were further along in the teacher education program were more likely to articulate the core idea in their explanations: 40% of pre-admitted students, 50% of students in Year 1, and 67% of students in Year 2 articulated the core idea.

CONCLUSIONS

- Preservice teachers can use written work and interaction with a simulated student to generalize about problem features needed for a strategy to work and identify scenarios in which the strategy would not work.
- Explaining when and why the strategy both will and will not work reveals different nuances in preservice teachers' thinking and facilitates distinctions between cases of superficial and deep understanding.
- Preservice teachers may experience challenges in generalizing about nonstandard approaches, such as:
  - Identifying mathematical ideas at the core of the strategy.
  - Using knowledge of properties of shapes to reason about how an area strategy might apply to a given problem.
  - Using accurate and precise mathematical language to describe concepts of geometry and measurement.
- Further investigation is needed to explore the relationship between program learning experiences and differences in preservice teachers' mathematical knowledge for teaching.

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