

INTERPRETING STUDENT THINKING: FEATURES OF AND CHANGES IN PRESERVICE TEACHERS' USE OF INFERENCES AND SUPPORTING EVIDENCE

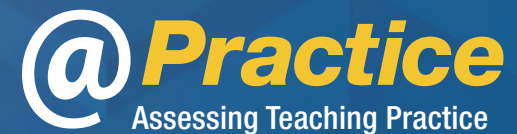
Meghan Shaughnessy, Timothy Boerst, & Rosalie DeFino

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ASSESSMENT MATTERS

Assessing is integral to teaching and therefor should be foundational to the preparation of teachers. (e.g. AMTE, 2017)

“Assessment is the act of gathering evidence about student knowledge or ability to use mathematics and make inferences from that evidence for a variety of purposes/audiences” (NCTM, 1995)

CONSIDERING A TEACHING PRACTICE ESSENTIAL TO ASSESSING: INTERPRETING STUDENT THINKING

Interpreting: Characterizing what a student thinks based on evidence from the student's words, actions, or writing

- Making qualified claims about valued outcomes that can be used as the basis for future action
- Using evidence to generate and test claims
- Matching the scope and nature of the claim to the amount and type of information available
- Actively working to prevent bias or distortion
- Developing or using appropriate criteria to focus or inform judgments

(Developed drawing on Stiggins, 2001)

FOCUSING ON INTERPRETING FROM THE BEGINNING OF TEACHER EDUCATION

Early attention to **interpreting student thinking** is crucial, because:

- people are likely to develop ways of doing this in everyday life
- errors in focus, scope and/or evidence are consequential for students' learning and life opportunities
- it is a rich territory in which to notice, and work to address/counteract, the impacts of bias

LEARNING ABOUT PRESERVICE TEACHERS' SKILL WITH INTERPRETING

Simulations are approximations of practice that can be used for assessing professional learning.

Simulations:

- are commonly used in many professional fields
- place authentic, practice-based demands on a participant
- purposefully suspend or standardize some elements of the practice-based situation
- can provide insights that are not possible or practical to determine in real-life professional contexts

COMING UP IN THIS SESSION

- An example of the simulation that served as the basis for learning about the preservice teachers' skills with interpretation
- Analysis of preservice teachers' interpretation of a student's mathematical thinking at the:
 - Baseline - simulation at the outset of the TE program
 - Year 1 - simulation at the midpoint in the TE program

THE PRESERVICE TEACHER PREPARES

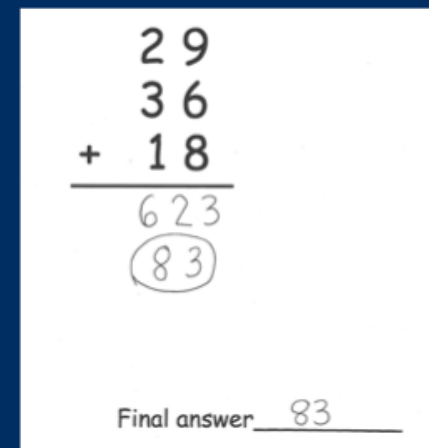
The preservice teacher:

1. prepares for an interaction with a standardized student about one piece of student work

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$

Final answer 83

Your goal is to elicit and probe to find out what the “student” did to produce the answer as well as the way in which the student understands the steps that were performed



The image shows a student's handwritten work for the addition problem 29 + 36 + 18. The numbers are stacked vertically: 29, 36, and 18. A horizontal line is drawn below 18. Below the line, the student has written 623. Below 623, the student has circled the number 83. At the bottom of the work, it says "Final answer 83".

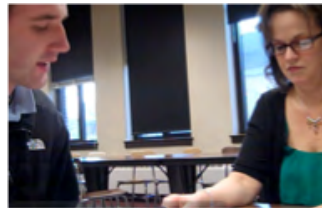
Correct answer, alternative algorithm, degree of understanding is unclear

THE PRESERVICE TEACHER ENGAGES IN A SIMULATION

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$

The preservice teacher:

1. prepares for an interaction with a standardized student about one piece of student work
2. interacts with the student to probe the standardized student's thinking



A Standardized Student

Developed response guidelines focused on:

- What the student is thinking such as
 - Uses an alternative algorithm (column addition), except the student is working from left to right
 - Applies the method correctly and has conceptual understanding of the procedure
- General orientations towards responses such as
 - Talk about digits in columns in terms of the place value of the column (e.g., 23 ones)
 - Give the least amount of information that is still responsive to the question
- Responses to anticipated questions

SAMPLE SIMULATION

On the basis of this interaction, what can be said about this student's process for solving this addition problem and her understanding of that process?



PRESERVICE TEACHER IS INTERVIEWED

The preservice teacher:

1. prepares for an interaction with a standardized student about one piece of student work
2. interacts with the student to probe the standardized student's thinking
3. **responds to questions about her/his interpretation of the student's thinking, including predicting the student's response on a similar task**

Interviewing about interpretations

Preservice teachers are asked to

- Describe the student's process
- Indicate what the student does and does not understand about the process

Preservice teachers are asked to apply what they learned to

- Anticipate how the student would solve a similar problem

$$\begin{array}{r} 27 \\ + 48 \\ \hline \end{array}$$

- Provide interpretations of understandings that are at the core of the process

INTERPRETING SKILLS OF PRESERVICE TEACHERS AT THE START OF THE PROGRAM



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INITIAL SKILL IN INTERPRETING STUDENT THINKING

Context:

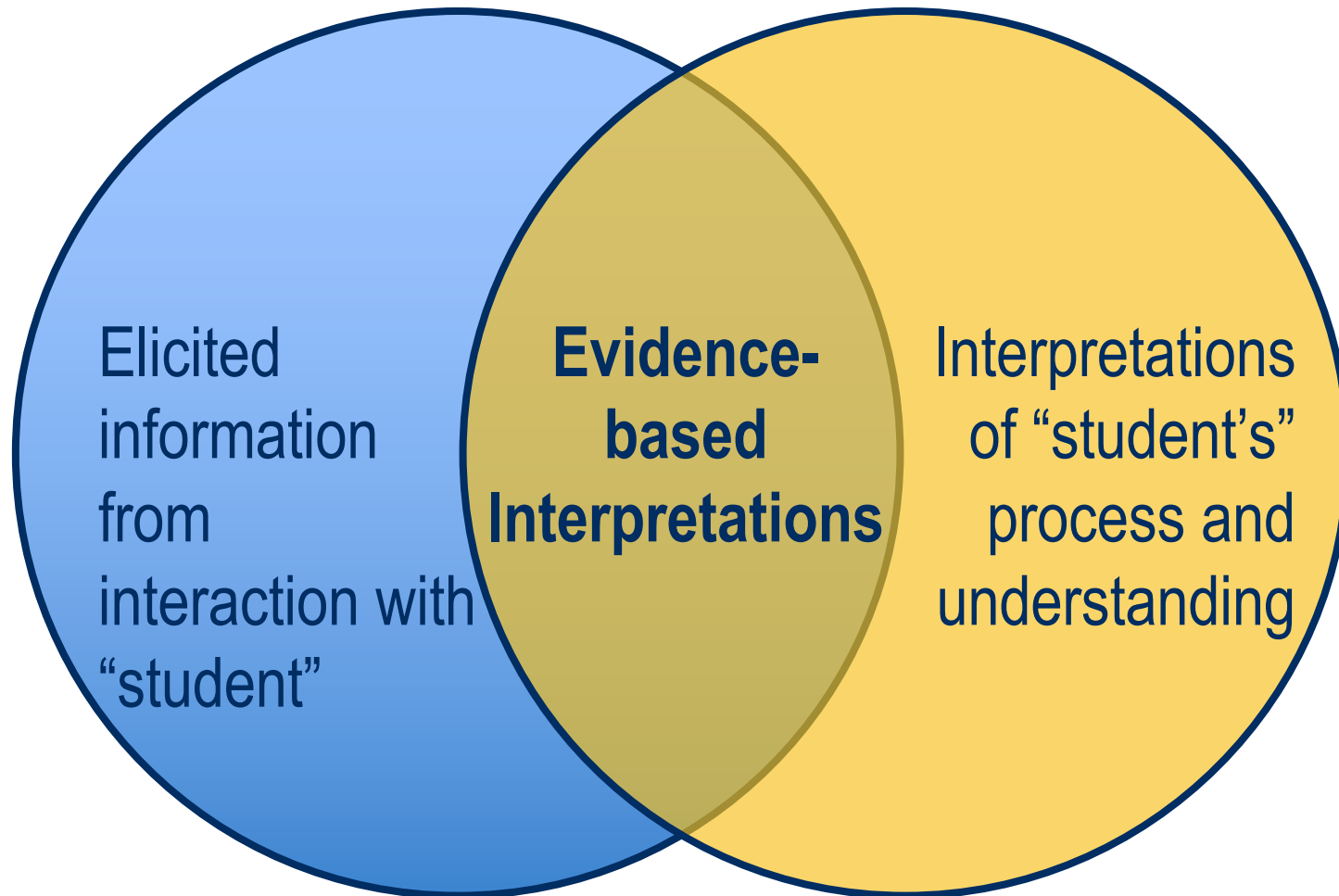
- Simulation assessment (46 preservice teachers)
- Data collected during the first week of the teacher education program

Analyzing the nature and prevalence of interpretations:

- focusing on mathematical process used by the student
- focusing on the student's understanding of the process
- anticipating method and understanding of work on a similar problem
- marshalling available evidence to support claims

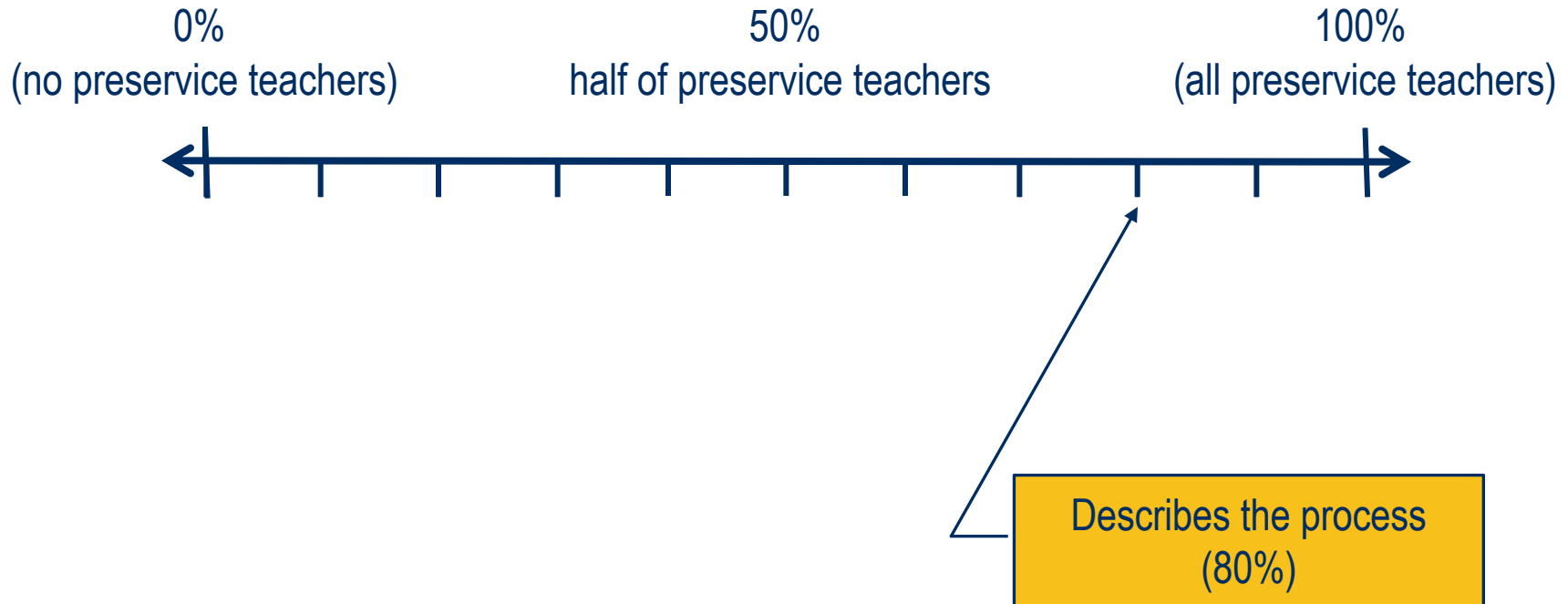


EVIDENCE-BASED INTERPRETATIONS



PREVALENCE OF INTERPRETATIONS

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \hline (83) \end{array}$$



PREVALENCE OF INTERPRETATIONS

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$

$$\begin{array}{r} 27 \\ + 48 \\ \hline \end{array}$$

Based on your interaction with the student, how do you think the student would solve this problem if the student used the same process?

83% of preservice teachers correctly anticipated the student's process

$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

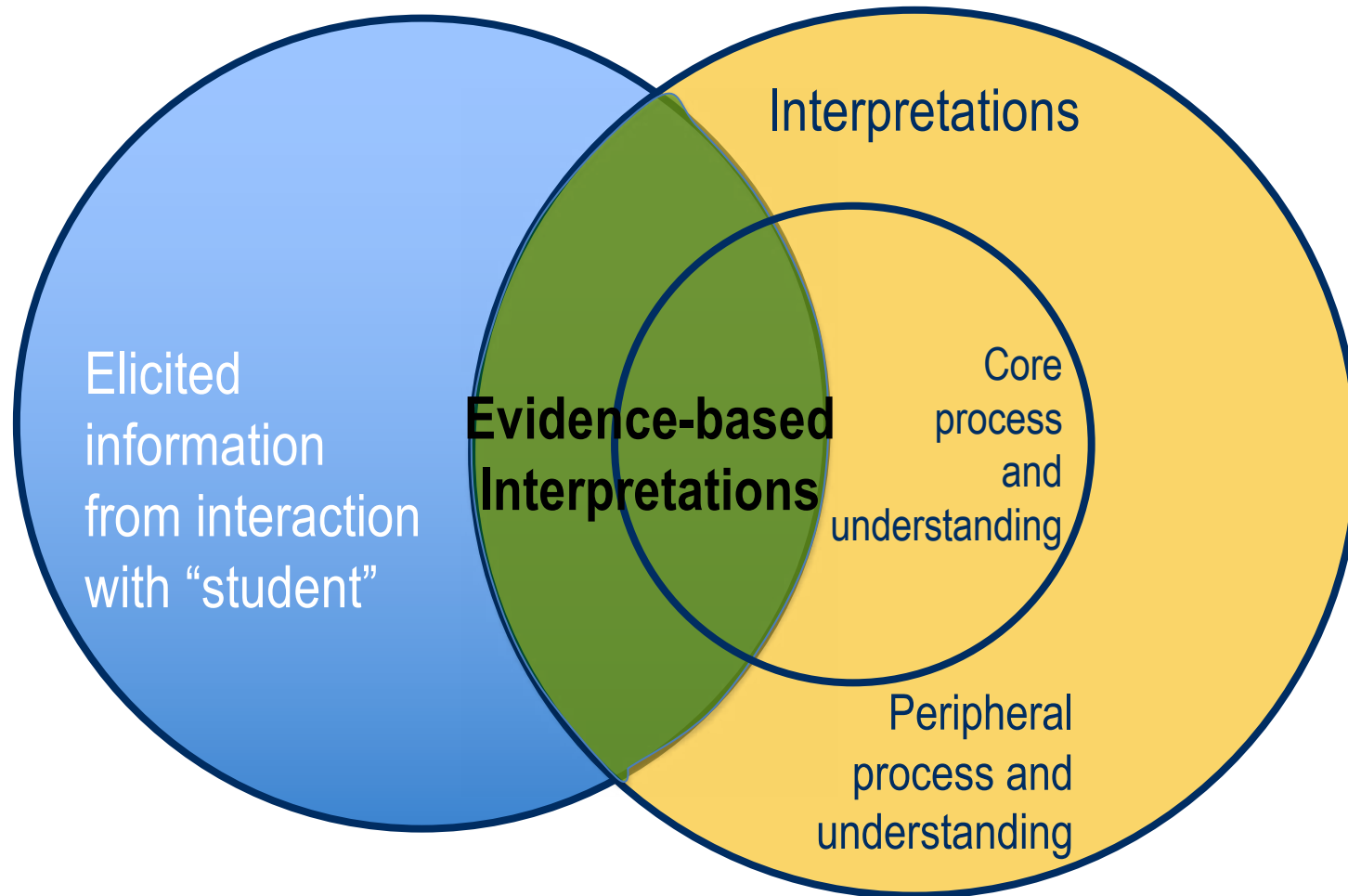
incorrect
sequence

$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ \textcircled{75} \end{array}$$

incorrect

$$\begin{array}{r} 27 \\ + 48 \\ \hline 6015 \\ 75 \end{array}$$

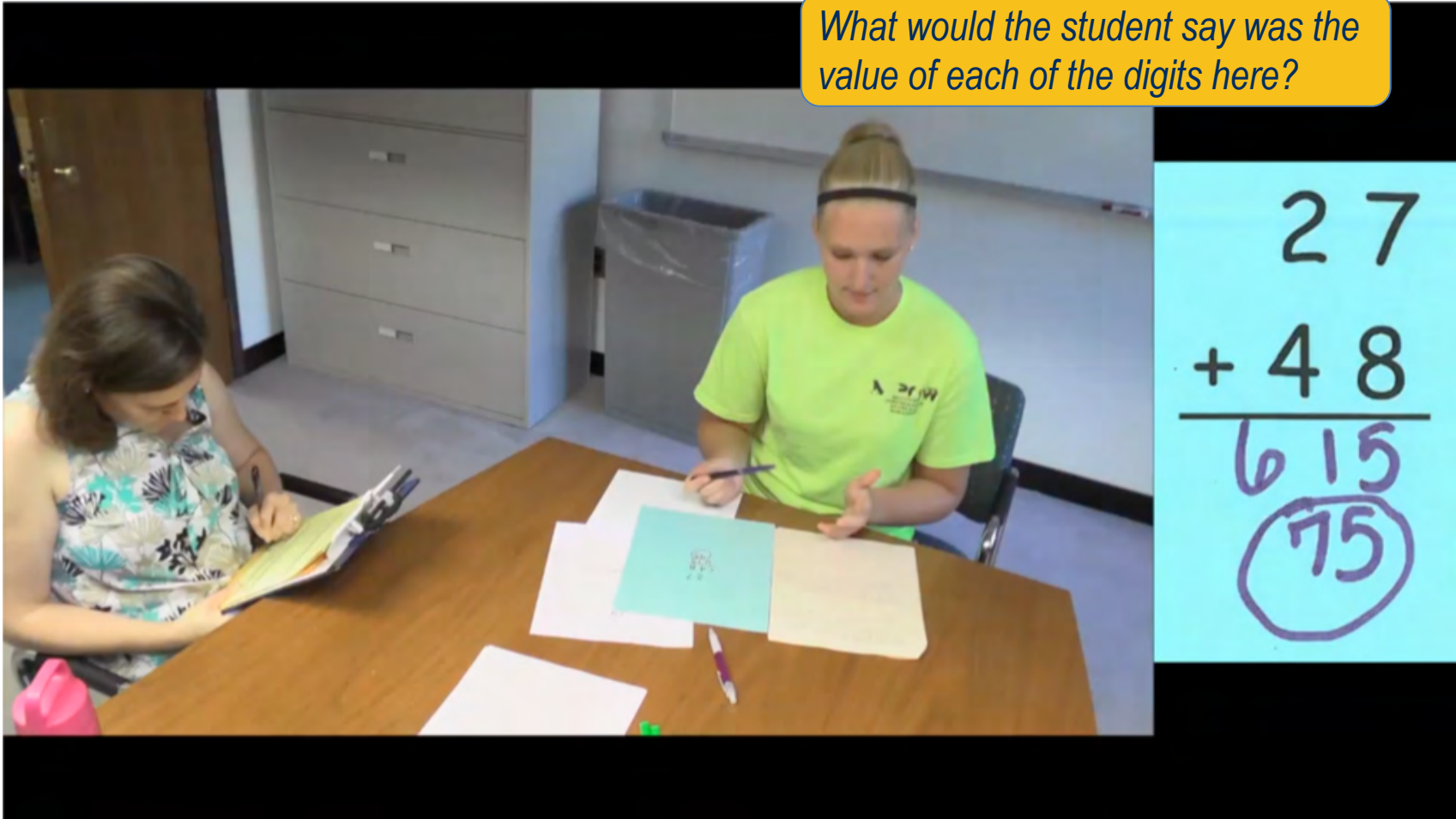
SOME TYPES OF INTERPRETATIONS



ANTICIPATES UNDERSTANDING

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$
$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

What would the student say was the value of each of the digits here?



ANTICIPATES UNDERSTANDING

$$\begin{array}{r}
 29 \\
 36 \\
 + 18 \\
 \hline
 623 \\
 \textcircled{83}
 \end{array}$$

$$\begin{array}{r}
 27 \\
 + 48 \\
 \hline
 615 \\
 75
 \end{array}$$

What would the student say was the value of each of the digits here?

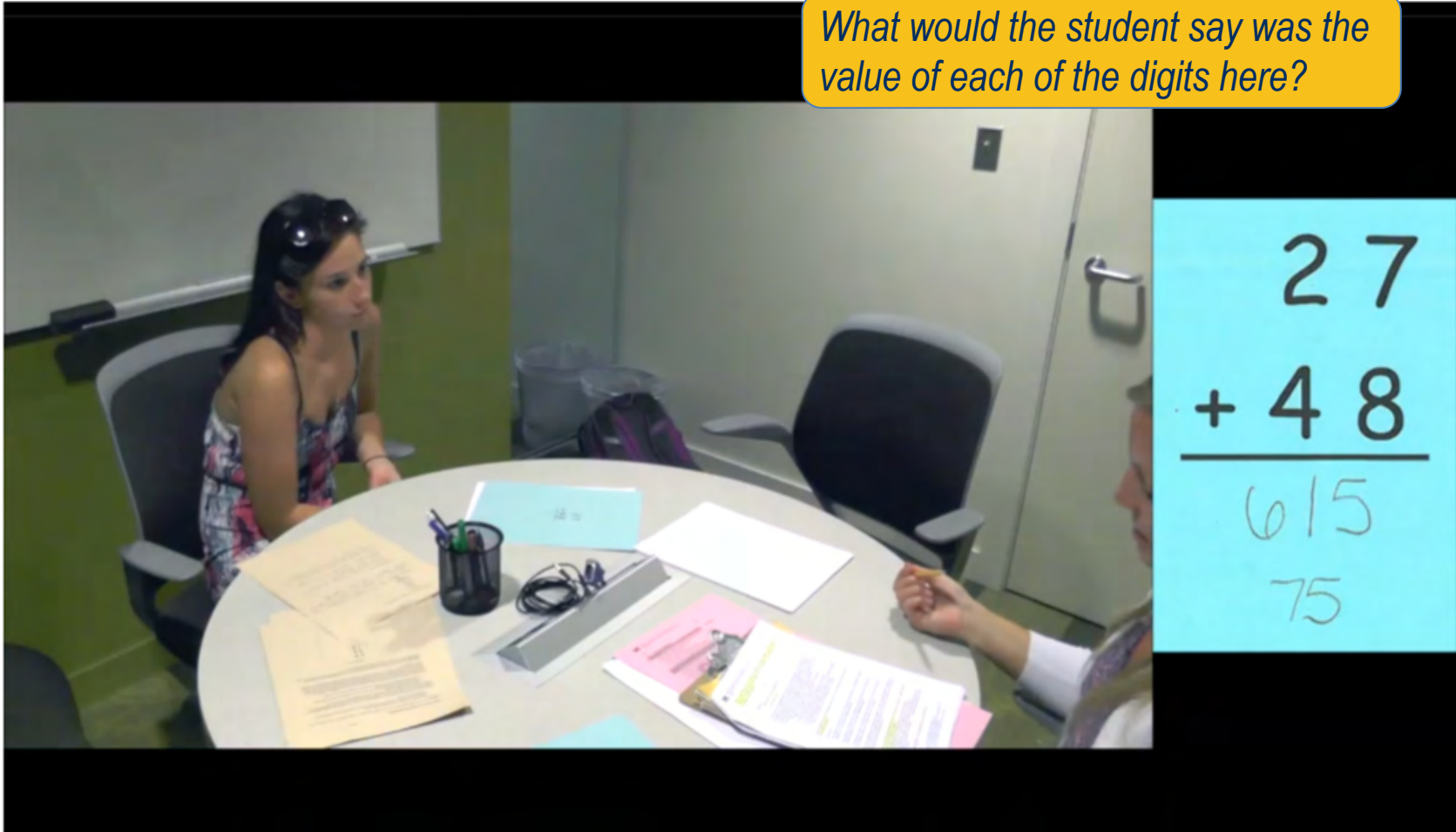
| | BASELINE |
|--|----------------------------------|
| | The Meaning of the Digits in 615 |
| Anticipates understanding with evidence | 24% |
| Recognizes the need for more information before anticipating understanding | 26% |
| Anticipates understanding incorrectly* | 50% |

* includes incorrect claims and making a claim without having gathered evidence

ANTICIPATES UNDERSTANDING

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$
$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

What would the student say was the value of each of the digits here?



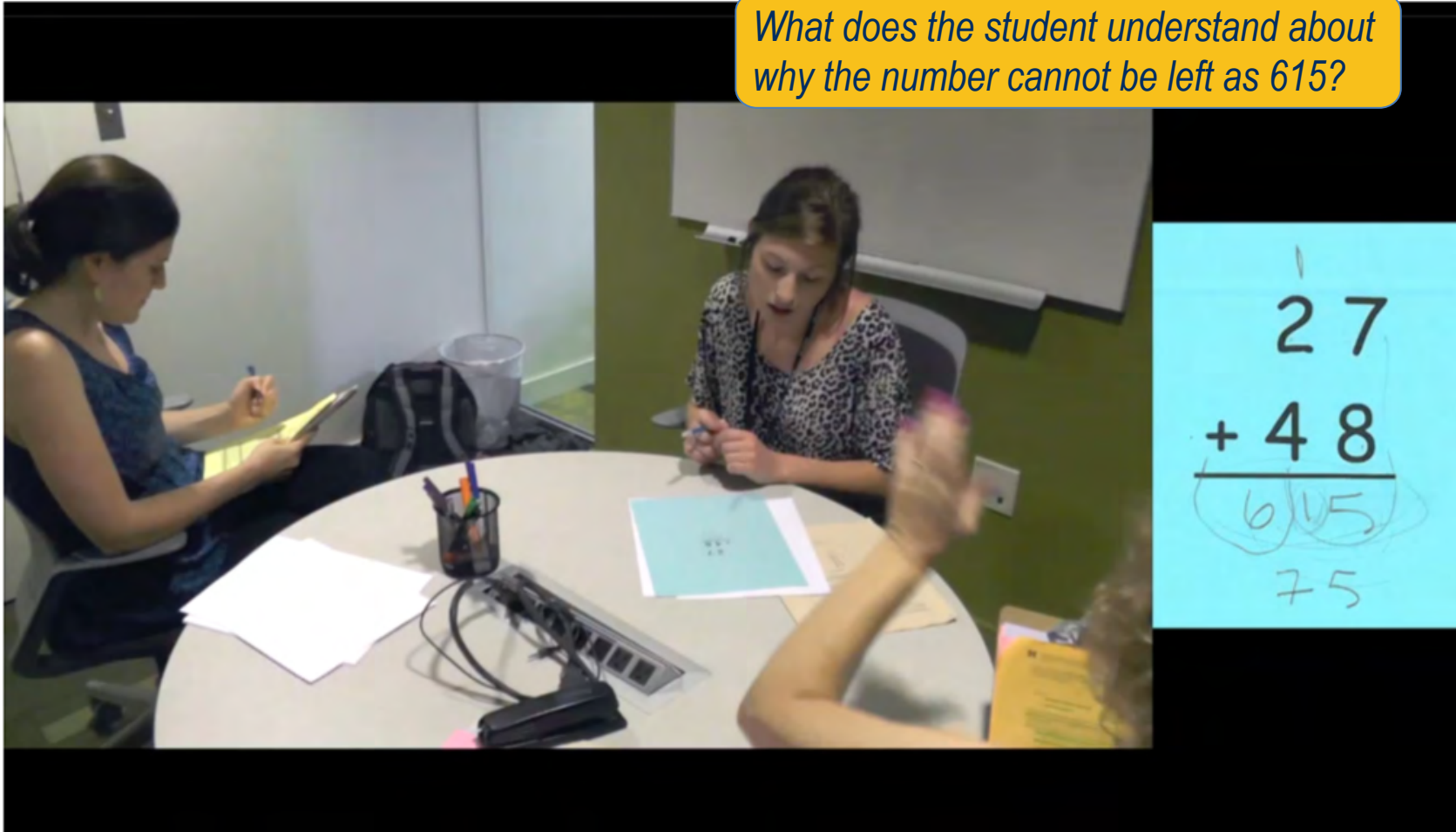
$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

ANTICIPATES UNDERSTANDING

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$

$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

What does the student understand about why the number cannot be left as 615?



$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

ANTICIPATES UNDERSTANDING

$$\begin{array}{r}
 29 \\
 36 \\
 + 18 \\
 \hline
 623 \\
 \textcircled{83}
 \end{array}$$

$$\begin{array}{r}
 27 \\
 + 48 \\
 \hline
 615 \\
 75
 \end{array}$$

What does the student understand about why the number cannot be left as 615?

| | BASELINE | |
|--|----------------------------------|--------------------------------------|
| | The Meaning of the Digits in 615 | Why the Number Cannot Be Left As 615 |
| Anticipates understanding with evidence | 24% | 63% |
| Recognizes the need for more information before anticipating understanding | 26% | 11% |
| Anticipates understanding incorrectly* | 50% | 26% |

* includes incorrect claims and making a claim without having gathered evidence

ANTICIPATES UNDERSTANDING

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$

$$\begin{array}{r} 27 \\ + 48 \\ \hline 615 \\ 75 \end{array}$$

| | BASELINE | |
|--|----------------------------------|--------------------------------------|
| TYPE OF INTERPRETING CONTEXT | SAFE TO ASSUME | PROVOKES A QUESTION |
| | The Meaning of the Digits in 615 | Why the Number Cannot Be Left As 615 |
| Anticipates understanding with evidence | 24% | 63% |
| Recognizes the need for more information before anticipating understanding | 26% | 11% |
| Anticipates understanding incorrectly* | 50% | 26% |

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SUPPORTING THE DEVELOPMENT OF SKILL WITH INTERPRETING



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MATH CONTENT COURSES

- Two 3 credit “mathematics for elementary teaching” courses
- Taught by faculty in the math department
- Content across mathematical strands
- Overall goals:
 - “Develop a deep understanding of the structure underlying the mathematical concepts that could be used to create instructive tasks for elementary school students and to understand and effectively utilize student thinking in the classroom.”
 - “Be able to look at elementary school student work and extract its mathematical content, even if it looks different than ‘textbook’ work.”

CHILDREN AS SENSE-MAKERS

- Two credit course focused on developing the following practices:
 - Eliciting and interpreting children’s mathematical thinking
 - Explaining mathematical content, with a focus on fractions
 - Using assessment information to inform instruction
- Taught by School of Education Faculty
- Follows a 4-week “Children as Sense-Makers” course, which focuses on eliciting and interpreting children’s thinking in the context of science
- Includes experiences interpreting students’ written work, video of students talking about mathematics, embedded work in a 5th grade classroom and in interns’ field placements (3rd – 5th grade classrooms)

INTERPRETING SKILLS OF PRESERVICE TEACHERS AT THE MID-POINT OF THE PROGRAM



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ASSESSING SKILLS OF ELICITING AND INTERPRETING STUDENT THINKING (AGAIN)

CONTEXT

- **Focus:** Eliciting and interpreting student thinking with particular mathematics content
- **Timing:** End of the first year in the program; after coursework focused on eliciting and interpreting student thinking

ASSESSMENT PROMPT

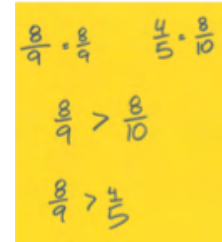
Which fraction is greater: $\frac{3}{7}$ or $\frac{2}{5}$

$$\frac{3}{7} = \frac{6}{14} \quad \frac{2}{5} = \frac{6}{15}$$

$$\frac{6}{14} > \frac{6}{15}$$

$$\text{So: } \frac{3}{7} > \frac{2}{5}$$

ANTICIPATES UNDERSTANDING



| | MID-PROGRAM |
|--|-------------------------|
| | Common Numerator Method |
| Anticipates understanding with evidence | 80% |
| Recognizes the need for more information before anticipating understanding | 9% |
| Anticipates understanding incorrectly* | 11% |

* includes incorrect claims and making a claim without having gathered evidence

ANTICIPATING THE STUDENT'S UNDERSTANDING OF THE COMMON NUMERATOR METHOD

$$\frac{2}{100} = \frac{2}{100} \quad \frac{1}{10} = \frac{10}{100}$$

$$\frac{2}{100} > \frac{10}{100}$$

$$\frac{2}{100} > \frac{1}{10}$$

The whiteboard shows the following work:

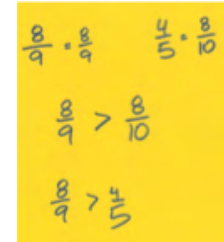
$$\frac{15}{35} = \frac{5 \times 3}{5 \times 7} > \frac{2 \times 7}{5 \times 1} = \frac{14}{35}$$

$$\frac{3}{7} = \frac{6}{14}$$

$$\frac{2}{5} = \frac{6}{15}$$

$$\frac{6}{14} > \frac{6}{15}$$

ANTICIPATES UNDERSTANDING



| | MID-PROGRAM | |
|--|-------------------------|---|
| | Common Numerator Method | Process For Generating Equivalent Fractions |
| Anticipates understanding with evidence | 80% | 20% |
| Recognizes the need for more information before anticipating understanding | 9% | 50% |
| Anticipates understanding incorrectly* | 11% | 30% |

* includes incorrect claims and making a claim without having gathered evidence

ANTICIPATING THE STUDENT'S UNDERSTANDING OF THE PROCESS FOR GENERATING EQUIVALENT FRACTIONS

$$\frac{3}{7} = \frac{6}{14} \quad \frac{2}{5} = \frac{4}{10}$$

$$\frac{6}{14} > \frac{4}{10}$$

$$\frac{3}{7} > \frac{2}{5}$$

Which fraction is greater: $\frac{3}{7}$ or $\frac{2}{5}$ $\frac{3}{7}$ or $\frac{2}{7}$

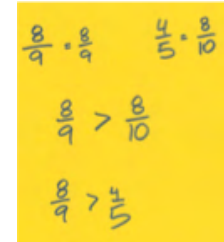
$$\frac{3}{7} = \frac{6}{14} \quad \frac{2}{5} = \frac{4}{10}$$

$$\frac{6}{14} > \frac{4}{10}$$

So: $\frac{3}{7} > \frac{2}{5}$

$$\frac{3}{6} \text{ or } \frac{2}{10}$$

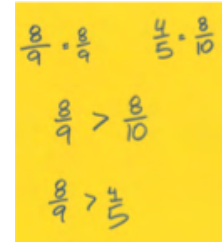
ANTICIPATES UNDERSTANDING



| | MID-PROGRAM | |
|--|-------------------------|---|
| | Common Numerator Method | Process For Generating Equivalent Fractions |
| Anticipates understanding with evidence | 80% | 20% |
| Recognizes the need for more information before anticipating understanding | 9% | 50% |
| Anticipates understanding incorrectly* | 11% | 30% |

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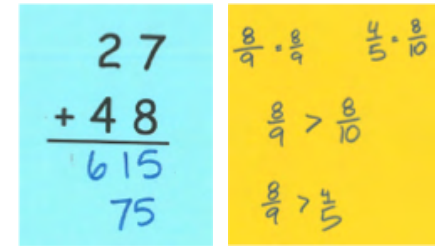
ANTICIPATES UNDERSTANDING



| | MID-PROGRAM | |
|--|-------------------------|---|
| TYPE OF INTERPRETING CONTEXT | PROVOKES A QUESTION | SAFE TO ASSUME |
| | Common Numerator Method | Process For Generating Equivalent Fractions |
| Anticipates understanding with evidence | 80% | 20% |
| Recognizes the need for more information before anticipating understanding | 9% | 50% |
| Anticipates understanding incorrectly* | 11% | 30% |

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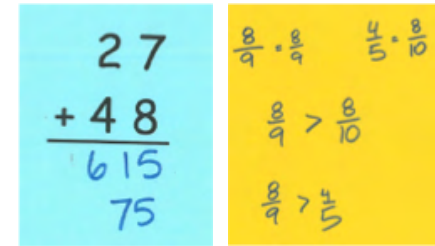
ANTICIPATES UNDERSTANDING



| | BASELINE | MID-PROGRAM |
|--|----------------------------------|---|
| TYPE OF INTERPRETING CONTEXT | SAFE TO ASSUME | SAFE TO ASSUME |
| | The Meaning of the Digits in 615 | Process For Generating Equivalent Fractions |
| Anticipates understanding with evidence | 24% | 20% |
| Recognizes the need for more information before anticipating understanding | 26% | 50% |
| Anticipates understanding incorrectly* | 50% | 30% |

* includes incorrect claims and making a claim without having gathered evidence

ANTICIPATES UNDERSTANDING



| | BASELINE | MID-PROGRAM |
|--|--------------------------------------|----------------------------|
| TYPE OF INTERPRETING CONTEXT | PROVOKES A QUESTION | PROVOKES A QUESTION |
| | Why the Number Cannot Be Left As 615 | Common Numerator Method |
| Anticipates understanding with evidence | 63% | 80% |
| Recognizes the need for more information before anticipating understanding | 11% | 9% |
| Anticipates understanding incorrectly* | 26% | 11% |

* includes incorrect claims and making a claim without having gathered evidence

CONCLUSIONS



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WHAT CAN BE LEARNED

This small study of preservice teachers in a TE program illustrates that:

1. Preservice teachers can use written work and interaction with a “student” in a simulation as the basis for generating evidence-based interpretations of student thinking
2. When interpreting student thinking in contexts that were mathematically “question provoking,” preservice teachers grew in their ability to use evidence to support correct interpretations
3. When interpreting student thinking in contexts that were mathematically “safe to assume,” preservice teachers grew in their awareness that they didn’t have sufficient evidence to make an interpretation

Lingering question: What kinds of experiences could support preservice teachers in learning to more actively question their assumptions about students’ understanding?