

DESIGNING ASSESSMENTS OF MATHEMATICS TEACHING PRACTICE: EXPLORING DESIGN RESOURCES FOR SIMULATIONS

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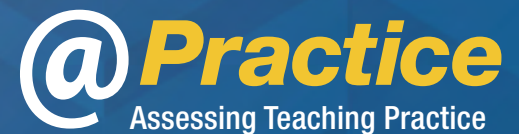
2018 MI-AMTE Conference: Conversations Among Colleagues

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WHAT DO ASSESSMENTS OF PRESERVICE TEACHERS' CAPABILITIES NEED TO BE LIKE?

- Assess entry-level practice: focus on skills and knowledge for the work of teaching
- Provide information about teacher candidates' development and about instructional needs
- Be useful to teacher candidates' and program administrators, and also demonstrate professional accountability and rigor to external stakeholders
- Use time efficiently and resources wisely

PREVAILING APPROACHES TO ASSESSING TEACHING PRACTICE

- Field observations
- Video recordings

But solely using these types of assessments is challenging....

USING STANDARDIZED SIMULATIONS TO ASSESS TEACHING PRACTICE

Simulations are approximations of practice that can be used for both assessing and supporting ongoing learning

Simulations:

- are commonly used in many professional fields
- place authentic, practice-based demands on a participant
- purposefully suspend or standardize some elements of the practice-based situation
- can provide information that are not possible or practical to determine in real-life professional context

ELICITING STUDENT THINKING

A high-leverage teaching practice: to find out what students know or understand, and how they are thinking/reasoning

- Launching an interaction with a student
- Asking follow-up questions to learn about a student's
 - Process for solving a mathematics problem
 - Understanding of mathematical ideas involved in a problem
- Being responsive to students by
 - Attending to and taking up student ideas
 - Maintaining a tone and manner that encourages the student to share their thinking

THE PRESERVICE TEACHER PREPARES

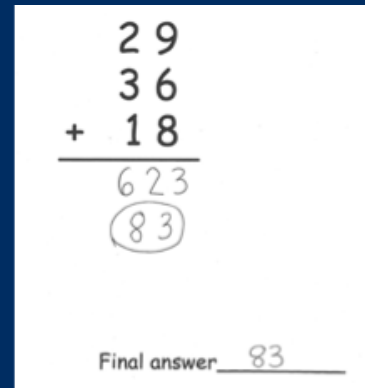
The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

$$\begin{array}{r} 29 \\ + 36 \\ \hline 623 \\ \textcircled{83} \end{array}$$

Final answer 83

Your goal is to elicit and probe to find out what the “student” did to produce the answer as well as the way in which the student understands the steps that were performed


$$\begin{array}{r} 29 \\ + 36 \\ \hline 623 \\ \textcircled{83} \end{array}$$

Final answer 83

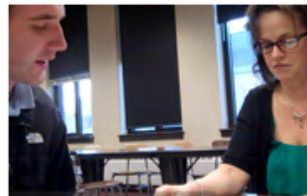
Correct answer, alternative algorithm, degree of understanding is unclear

THE PRESERVICE TEACHER ENGAGES IN A SIMULATION

$$\begin{array}{r} 29 \\ 36 \\ + 18 \\ \hline 623 \\ \textcircled{83} \end{array}$$

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work
2. **Interacts with the student to probe the standardized student's thinking**



A Standardized Student

Developed response guidelines focused on:

- What the student is thinking such as
 - Uses an alternative algorithm (column addition), except the student is working from left to right
 - Applies the method correctly and has conceptual understanding of the procedure
- General orientations towards responses such as
 - Talk about digits in columns in terms of the place value of the column (e.g., 23 ones)
 - Give the least amount of information that is still responsive to the question

THE PRESERVICE TEACHER IS INTERVIEWED

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work
2. Interacts with the student to probe the standardized student's thinking
3. **Responds to questions about her/his interpretation of the student's thinking, including predicting the student's response on a similar task**

Interviewing about interpretations

Preservice teachers are asked to

- Describe the student's process
- Indicate what the student does and does not understand about the process

Preservice teachers are asked to apply what they learned to

- Anticipate how the student would solve a similar problem
- Provide interpretations of understandings that are at the core of the process

$$\begin{array}{r} 27 \\ + 48 \\ \hline \end{array}$$

DESIGNING SIMULATIONS OF MATHEMATICS TEACHING PRACTICE



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DESIGN COMPONENTS

Simulation assessment design requires:

- A sample of student work on a mathematics problem
- Detailed and believable student dialog/representation
- Knowledge of preservice teachers
- Ability to facilitate, collect evidence, and evaluate the teaching practice
- ...

Each of these requirements could be broken down further
assessment design is a challenging and complex undertaking

EXAMPLE DESIGN SPACE

Parameters:

- Primary grade
- Number and operation
- “Word problem” work
- Participants in Elementary TE
- Logistics and routines similar to those of other simulation assessments

Simulation assessment design requires:

- A sample of student work on a mathematics problem
- Detailed and believable student dialog/representation
- Knowledge of preservice teachers
- Ability to facilitate, collect evidence, and evaluate the teaching practice

DESIGN RESOURCES

Teacher educators' wisdom of practice is crucial....

...*But likely not enough to generate needed assessments*

Simulation assessment design can be support using a range of resources, including but not limited to:

- Elementary Mathematics curriculum materials
- Mathematics education research – student thinking, preservice teacher knowledge and skills,
- Cases of students working on math problems

DESIGN RESOURCES: RESEARCH

Research provided:

- Examples and categories of word problems

(Carpenter, Fennema, Franke, Levi, & Empson, 1999)

Problem

Join (Result Unknown)

Ellen had 3 tomatoes. She picked 9 more tomatoes. How many tomatoes does Ellen have now?

Join (Change Unknown)

Chuck has 3 dollars. How many more dollars does he need to buy a stuffed animal that costs 12 dollars?

Separate (Result Unknown)

There were 12 seals playing. 9 seals swam away. How many seals were still playing?

Separate (Change Unknown)

There were 12 people on the bus. Some people got off. Now there are 3 people on the bus. How many people got off the bus?

Compare (Difference Unknown)

Megan has 3 stickers. Randy has 12 stickers. How many more stickers does Randy have than Megan?

Join (Start Unknown)

Deborah had some books. She went to the library and got 9 more books. Now she has 12 books altogether. How many books did she have to start with?

DESIGN RESOURCES: RESEARCH

Research provided:

- Examples and categories of word problems
- Descriptions of patterns of student thinking

Direct Modeling Strategies	
Problem	Strategy Description
<i>Join (Result Unknown)</i> Ellen had 3 tomatoes. She picked 9 more tomatoes. How many tomatoes does Ellen have now?	<i>Joining All</i> The child constructs a set of 3 objects and a set of 9 objects. He finds the answer by counting all the objects in the two sets.
<i>Join (Change Unknown)</i> Chuck has 3 dollars. How many more dollars does he need to buy a stuffed animal that costs 12 dollars?	<i>Joining To</i> The child constructs a set of 3 objects. She adds objects to this set until there is a total of 12 objects. She finds the answer by counting the number of objects added.
<i>Separate (Result Unknown)</i> There were 12 seals playing. 9 seals swam away. How many seals were still playing?	<i>Separating From</i> The child constructs a set of 12 objects and then removes 9 objects. She finds the answer by counting the remaining objects.
<i>Separate (Change Unknown)</i> There were 12 people on the bus. Some people got off. Now there are 3 people on the bus. How many people got off the bus?	<i>Separating To</i> The child counts out a set of 12 objects. He removes objects from the set until the number of objects remaining is equal to 3. He finds the answer by counting the objects he removed.
<i>Compare (Difference Unknown)</i> Megan has 3 stickers. Randy has 12 stickers. How many more stickers does Randy have than Megan?	<i>Matching</i> The child makes a set of 3 objects and a set of 12 objects. The two sets are matched one-to-one until one set is used up. She finds the answer by counting the unmatched objects remaining in the larger set.
<i>Join (Start Unknown)</i> Deborah had some books. She went to the library and got 9 more books. Now she has 12 books altogether. How many books did she have to start with?	<i>Trial and Error</i> The child constructs a set of objects. He adds a set of 9 objects to the set, and counts the objects in the resulting set. If the final count is 12, then the number of objects in the initial set is the answer. If it is not 12, he tries a different initial set.

Direct Modeling → Counting Strategies → Number Facts

(Carpenter, Fennema, Franke, Levi, & Empson, 1999)

DESIGN RESOURCES: RESEARCH

Research provided:

- Examples and categories of word problems
- Descriptions of patterns of student thinking

<i>Addition and Subtraction: Children's Solution Strategies</i>	15
Direct Modeling Strategies	15
Counting Strategies	18
Distinction Between Counting and Modeling Strategies	22
Number Facts	24
Relation of Strategies to Problem Types	25
Levels of Development of Strategies	26
<i>Direct Modeling Strategies</i>	26
<i>Counting Strategies</i>	28
<i>Flexible Choice of Strategies</i>	28
<i>Number Facts</i>	29
Integration of Solution Strategies and Problem Types	30

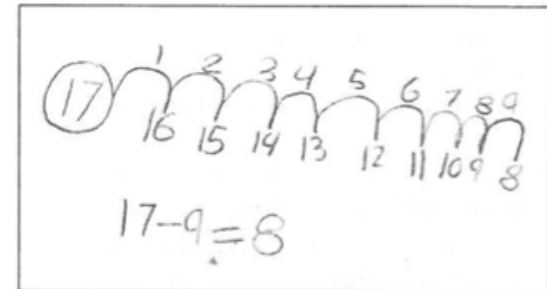
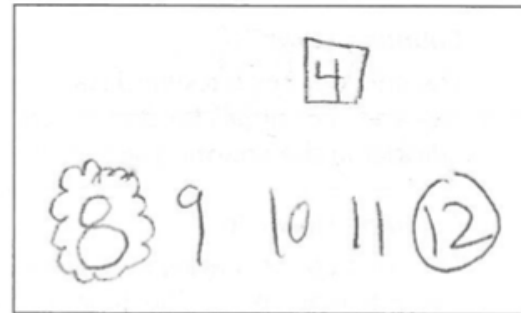
Direct Modeling → Counting Strategies → Number Facts

(Carpenter, Fennema, Franke, Levi, & Empson, 1999)

DESIGN RESOURCES: RESEARCH

Research provided:

- Examples and categories of word problems
- Descriptions of patterns of student thinking
- Examples of students employing strategies



(Carpenter, Fennema, Franke, Levi, & Empson, 1999)

DESIGN RESOURCES: CURRICULUM MATERIALS

Curriculum provided:

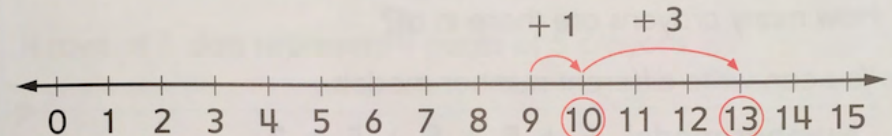
- Categories of approaches to solve problems
- Multiple descriptions of approaches
- Images of using representations

Counting-Up Subtraction

Start with the subtrahend and decide by how much you want to count up first. Count up, recording the “count-up” amount. Continue counting up until you reach the minuend. Then, to find the difference between the subtrahend and the minuend, find the total of all the count-up amounts.

You can also count up to find the difference.
Start with the smaller number.
Count up to the larger number.
The amount you count up is the difference.

Number model: $9 + ? = 13$



$$1 + 3 = 4$$

Number model: $9 + 4 = 13$

Jim is 4 years older than Ron.

DESIGN RESOURCES: CASES

Solve and consider how a 1st grade student might approach the following problem:

Mia scored 5 goals. Garret scored 14 goals. How many few goals did Mia score than Garret?

Comparison – Difference Unknown (CGI)

DESIGN RESOURCES: CASES

How could this resource support simulation design?

What challenges might this resource present for simulation design?

DESIGN RESOURCES: CASES

Cases provided:	<ul style="list-style-type: none">▪ Samples of student work▪ Specific, detailed examples of student talk and representation▪ Responses to questions tailored to reflect those preservice teacher might ask
Challenges for administrators:	<ul style="list-style-type: none">▪ Authentic dialog can be hard to replicate/standardize▪ Authentic representations can be challenging to produce▪ Path of student thinking might not be practical to elicit in the allocated time frame
Challenges for preservice teachers:	<ul style="list-style-type: none">▪ Student talk and representation might be contradictory or vague▪ Student talk and representation might be convoluted making it hard to interpret or know what to ask

PRODUCT OF THE DESIGN



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THE STUDENT'S PROCESS

1. "Underline the question and circle the information. 'How many' is information also."

Mia scored 5 goals. Garret scored 14 goals. How many fewer goals did Mia score than Garret?

THE STUDENT'S PROCESS

1. "Underline the question and circle the information. 'How many' is information also."
2. "So I write all the numbers. I do 5 to 14."

Mia scored 5 goals. Garret scored 14 goals. How many fewer goals did Mia score than Garret?

5 6 7 8 9 10 11 12 13 14

THE STUDENT'S PROCESS

1. "Underline the question and circle the information. 'How many' is information also."
2. "So I write all the numbers. I do 5 to 14."
3. "I counted from 5 and it was 9 steps to get to 14."

Mia scored 5 goals. Garret scored 14 goals. How many fewer goals did Mia score than Garret?

5 6 7 8 9 10 11 12 13 14

THE STUDENT'S PROCESS

1. "Underline the question and circle the information. 'How many' is information also."
2. "So I write all the numbers. I do 5 to 14."
3. "I counted from 5 and it was 9 steps to get to 14."
4. "I wrote the fact family. 5 plus 9 equals 14. 14 minus 9 equals 5."

Mia scored 5 goals. Garret scored 14 goals. How many fewer goals did Mia score than Garret?

5 6 7 8 9 10 11 12 13 14

$$5 + 9 = 14$$

$$14 - 9 = 5$$

$$14 - 9 = 5$$

THE STUDENT'S PROCESS

1. "Underline the question and circle the information. 'How many' is information also."
2. "So I write all the numbers. I do 5 to 14."
3. "I counted from 5 and it was 9 steps to get to 14."
4. "I wrote the fact family. 5 plus 9 equals 14. 14 minus 9 equals 5."
5. "I wrote 'Garret scored 9 more goals'."

Mia scored 5 goals. Garret scored 14 goals. How many fewer goals did Mia score than Garret?

5 6 7 8 9 10 11 12 13 14

$$5 + 9 = 14$$

14

$$14 - 9 = 5$$

Garret scored
9 more goals.

Mia scored 5 goals. Garret scored 14 goals. How many fewer goals did Mia score than Garret?

5 6 7 8 9 10 11 12 13 14

THE STUDENT'S UNDERSTANDING OF THE PROBLEM AND THE COUNTING UP METHOD

The student is thinking about the problem as an addition problem because it requires figuring out how much needs to be **added** to the smaller number to get the larger number

The student says, "I thought that 'how many fewer' was an addition problem because it says 'fewer'."

The student understands that the amount counted up represents the difference between Mia and Garret's goals, and that each "step" represents a goal

The student says, "I was trying to figure out how many more Mia needs to get to Garret" and "each [step] is a goal."

THE STUDENT'S UNDERSTANDING: FACT FAMILY AND LABEL FOR THE ANSWER

$5 + \square = 14$
 $14 - \square = 5$
 Garret scored
 9 more goals.

<p>The student knows that there are fact families and records a fact family with the numbers in the problem but does not use the equations in the fact family to solve the problem or check the answer</p>	<p><i>The student says, "I wrote the fact family... because you're supposed to."</i></p>
<p>The student understands that "Garret scored 9 more goals" is equivalent to "Mia scored 9 fewer goals."</p>	<p><i>The student says, "[Garret scored 9 more goals] is the same as Mia scoring 9 fewer."</i></p>

567891011121314
5-4=19
1-4=5
Garret scored
9 more goals.

OTHER INFORMATION ABOUT THE STUDENT'S THINKING IN THIS SCENARIO

The standardized student...

- Knows that word problems like this can be solved in multiple ways but prefers to use this strategy.
- Uses the phrases “counting on”, “number line”, and “steps” when describing his/her strategy.
- Does not yet know the traditional algorithms for multi-digit addition/subtraction.

CONCLUDING IDEAS



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CONCLUSION

To develop necessary components for simulation assessments, such as:

- A sample of student work on a mathematics problem
- Detailed and believable student dialog/representation
- Knowledge of preservice teachers
- Ability to facilitate, collect evidence, and evaluate the teaching practice...

Multiple resources can be of use:

- Elementary Mathematics curriculum materials
- Mathematics education research – student thinking, preservice teacher knowledge and skills,
- Cases of students working on math problems

Each with limitations that may be addressed by other resources...

But still requiring the wisdom of practice that mathematics teacher educators can bring to bear