

# DESIGNING SIMULATIONS TO ADVANCE PRESERVICE TEACHERS' KNOWLEDGE AND SKILLS

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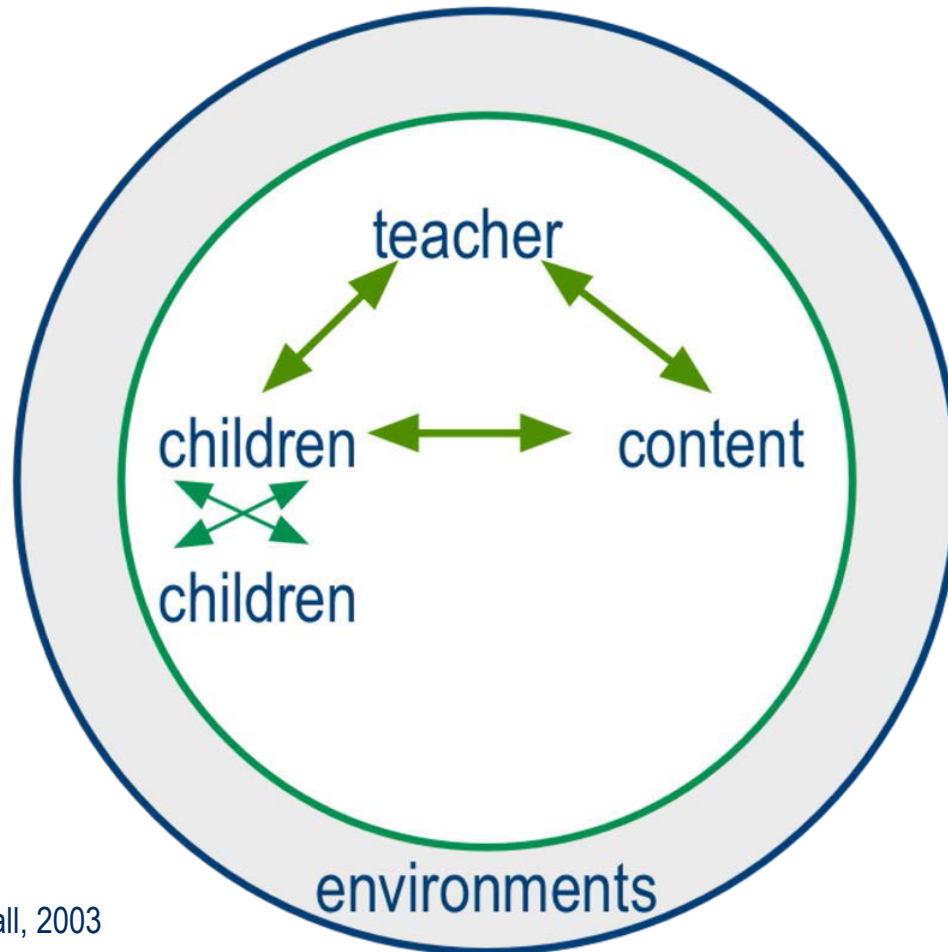
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# TEACHING AS INTERACTION



Cohen, Raudenbusch, & Ball, 2003

# MATHEMATICAL KNOWLEDGE FOR TEACHING

- Mathematical knowledge as it is used in the work of teaching is crucial for many tasks, including:
  - Choosing examples
  - Developing follow-up problems
  - Explaining ideas to students and supporting explanations
  - Using representations and materials to show the meaning of mathematical ideas

- Posing questions
- Interpreting students' thinking and choosing what to say or do in response

# ELICITING STUDENT THINKING

**A core teaching practice:** to find out what students know or understand, and how they are thinking/reasoning

- Establishing an environment in which a student is comfortable sharing his/her thinking
- Posing questions to get students to talk
- Listening to and hearing what students say
- Probing students' responses
- Developing an idea of what a student thinks
- Checking one's interpretation

# INTERPRETING STUDENT THINKING

Characterizing what a student thinks based on evidence from the student's words, actions, or writing

- Making qualified claims about valued outcomes that can be used as the basis for future action
- Using evidence to generate and test claims
- Matching the scope and nature of the claim to the amount and type of information available
- Actively working to prevent bias or distortion
- Developing or using appropriate criteria to focus or inform judgments

*(Developed drawing on Stiggins, 2001)*

# USING PRACTICE-BASED CONTEXTS TO DEVELOP USABLE MATHEMATICAL KNOWLEDGE

- Teachers' skill in being able to elicit and interpret student thinking is critical for supporting learning
- Mathematical Knowledge for Teaching is used when, **and can be developed through**, eliciting and interpreting student thinking

# SUPPORTING THE LEARNING OF MKT

Many methods can and have been used to support the learning of Mathematical Knowledge for Teaching, including:

- Analyzing video of students solving mathematics problems
- Analyzing and providing mathematics focused feedback on student work samples and planning a set of questions to ask a student
- Studying lessons, units, and trajectories in mathematics curriculum materials
- Constructing mathematics problems for students to solve
- Rehearsing the use of manipulatives to explain a mathematical process

# USING SIMULATIONS TO SUPPORT LEARNING

## Simulations:

- are approximations of practice
- are commonly used in many professional fields
- place authentic, practice-based demands on a participant
- purposefully suspend or standardize some elements of the practice-based situation
- can provide preservice teachers with insights that are not possible or practical to generate in real-life professional contexts



# WORKSHOP OVERVIEW

- Background: Using Simulations to Assess MKT
- Using insights from assessments to design simulations for developing MKT
  - Pre-simulation design space
  - During simulation design space
  - Post-simulation design space

# BACKGROUND: USING A SIMULATION TO ASSESS MKT



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# THE PRESERVICE TEACHER PREPARES FOR A SIMULATION

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

784  
- 315  
-----

Add 10 ones

Your goal is to elicit and probe to find out what the “student” did to produce the answer as well as the way in which the student understands the steps that were performed.

$$\begin{array}{r} 78\overset{14}{\cancel{4}} \\ - 3\overset{2}{\cancel{1}}5 \\ \hline 469 \end{array}$$

Correct answer, alternative algorithm, degree of understanding is unclear

# THE PRESERVICE TEACHER PREPARES FOR A SIMULATION

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work

How can the difference between the two numbers be re-established?

$$\begin{array}{r} 784 \\ - 315 \\ \hline \end{array}$$

Your goal is to elicit and probe to find out what the “student” did to produce the answer as well as the way in

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

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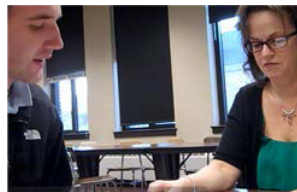
Correct answer, alternative algorithm, degree of understanding is unclear

# THE PRESERVICE TEACHER ENGAGES IN A SIMULATION

$$\begin{array}{r} 78\cancel{4}^{14} \\ - 3\cancel{1}5 \\ \hline 469 \end{array}$$

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work
2. **Interacts with the student to probe the standardized student's thinking**



## A Standardized Student

Developed response guidelines focused on:

- What the student is thinking such as
  - Uses a method not conventional in the U.S. (but that is standard in many European and South American countries)
  - Applies the method correctly and has conceptual understanding of the procedure
- General orientations towards responses such as
  - Talk about digits in columns in terms of the place value of the column (e.g., 14 ones)
  - Give the least amount of information that is still responsive to the question
- Responses to anticipated questions

# WHAT OPPORTUNITIES EXIST TO ASSESS THE PRESERVICE TEACHER'S MKT?



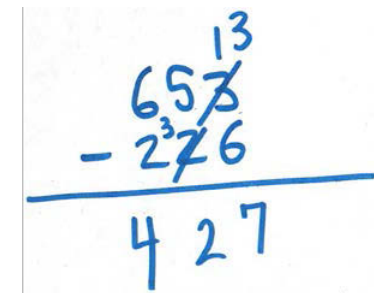


# WHAT OPPORTUNITIES EXIST TO ASSESS THE PRESERVICE TEACHER'S MKT?

$$\begin{array}{r} 78\cancel{4}^{14} \\ - 3\cancel{2}15 \\ \hline 469 \end{array}$$

The preservice teacher:

- Probes mathematics that is crucial for understanding the method
  - Does the student understand why adding 10 ones to the minuend and 1 ten to the subtrahend results in the same difference?
- Poses an additional task that is useful for confirming the student's method


$$\begin{array}{r} 65\cancel{7}^{13} \\ - 2\cancel{7}6 \\ \hline 427 \end{array}$$



# THE PRESERVICE TEACHER IS INTERVIEWED

The preservice teacher:

1. Prepares for an interaction with a standardized student about one piece of student work
2. Interacts with the student to probes the standardized student's thinking
3. **Responds to questions about her/his interpretation of the student's thinking, including predicting the student's response on a similar task**

## Interviewing about interpretations

Preservice teachers are asked to

- Summarize the student's process
- Indicate what the student does and does not understand about the process
- Generate a problem that could be used to confirm the student's process
- Anticipate how the student would solve a similar problem

$$\begin{array}{r} 761 \\ - 342 \\ \hline \end{array}$$

- Generalize whether the process will always work and why or why not

# OPPORTUNITIES TO SEE MKT IN USE

When we assess beginning teachers' skills with eliciting and interpreting a student's thinking, we can learn about their use of MKT as they —

- focus on specific elements of the mathematics in the problem
- word particular questions
- figure out additional problem(s) to probe the student's thinking
- predict how a student might approach a similar problem

# USING SIMULATIONS TO SUPPORT THE DEVELOPMENT OF MKT



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# TAKING CUES FOR DESIGN POSSIBILITIES FROM SIMULATION ASSESSMENTS



# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: PRE-SIMULATION

$$\begin{array}{r} 78 \quad 14 \\ 3 \quad 2 \quad 5 \\ \hline 469 \end{array}$$

Wonder? Yes... 2 guesses  
it works

$$\begin{array}{r} 5712 \\ - 174 \\ \hline 408 \end{array}$$

Across zero  

$$\begin{array}{r} 600 \\ - 13 \\ \hline 587 \end{array}$$
  
 even works  
 across zeros

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

5712  
 - 174  
 -----  
 difference is the same.  
 does student know this?

# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: PRE-SIMULATION

How could the student work examples from the simulation be used to provide opportunities to develop MKT?

- Provide approaches that are less familiar
- Provide approaches that lead to correct/incorrect answers
- Provide numerical examples that are special cases
- Provide more than one example
- ...

# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: PRE-SIMULATION

What would the following examples provide an opportunity to use or develop in terms of MKT?

$$\begin{array}{r} 784 \\ - 315 \\ \hline 469 \end{array}$$

$$\begin{array}{r} 732 \\ - 256 \\ \hline \end{array} \rightarrow \begin{array}{r} 700 + 30 + 2 \\ 300 + 60 + 6 \\ \hline 400 + 70 + 6 = 476 \end{array}$$

# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: PRE-SIMULATION

$$\begin{array}{r}
 784 \\
 - 315 \\
 \hline
 469
 \end{array}$$

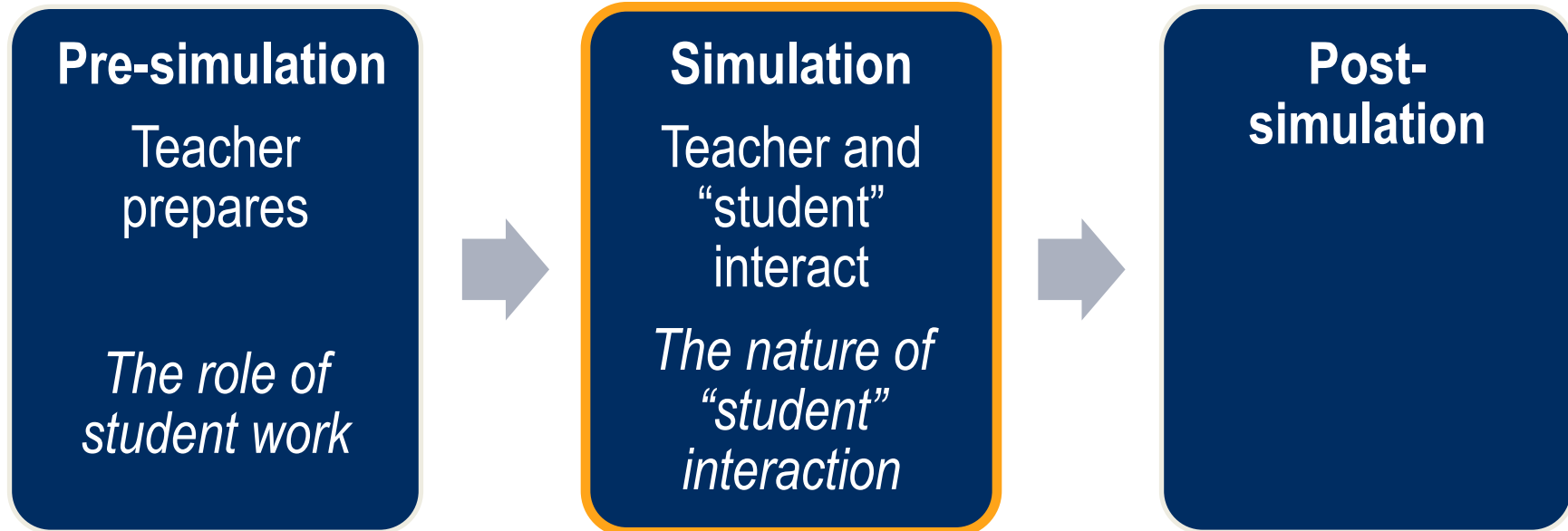
$$\begin{array}{r}
 732 \rightarrow 700 + 30 + 2 \\
 - 256 \rightarrow 300 + 60 + 6 \\
 \hline
 400 + 70 + 6 = 476
 \end{array}$$

You may have noticed one or more of the following:

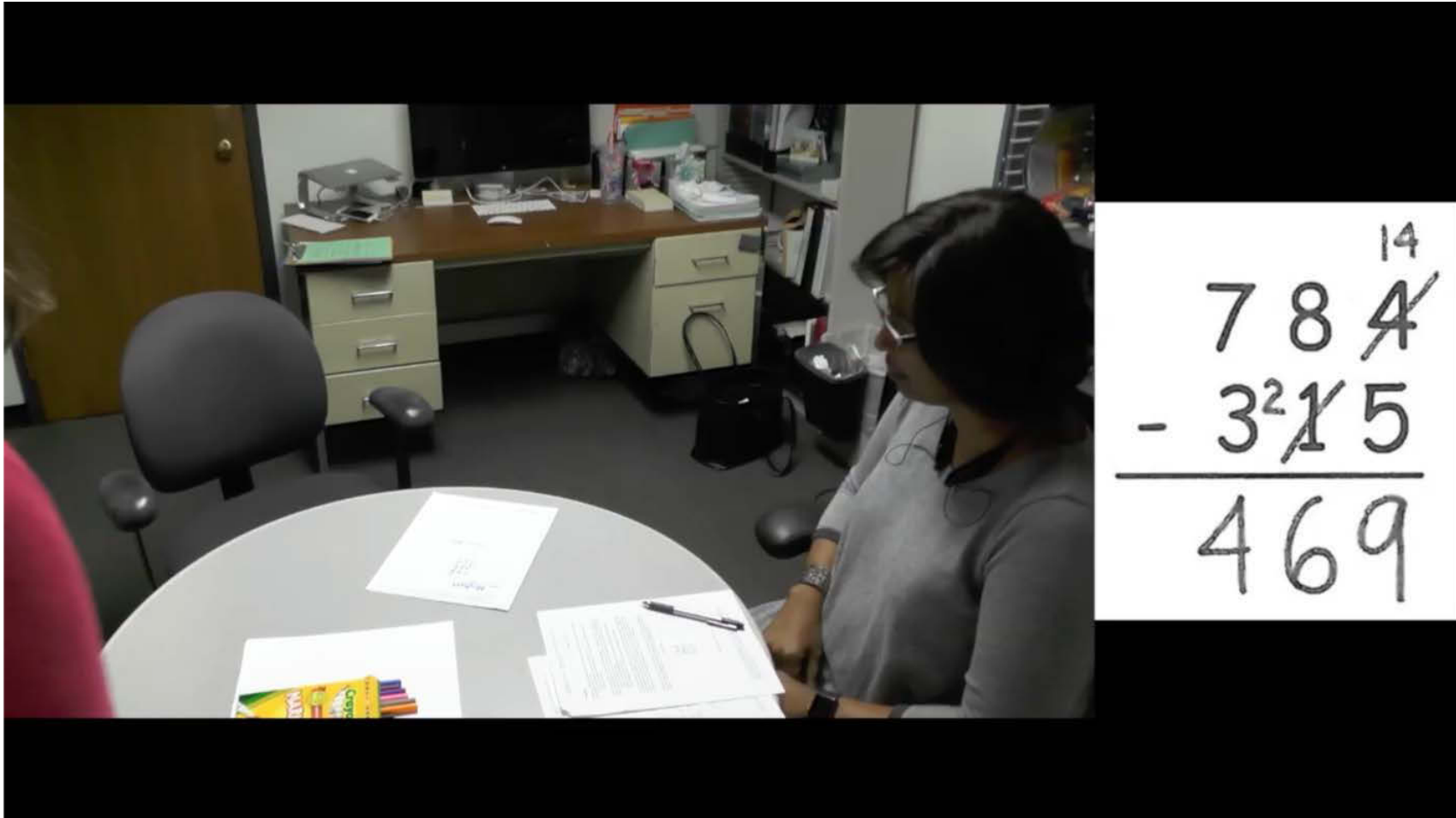
- the student can use more than one approach accurately
- the student uses the “same change” idea in both processes
- the blue method is more transparent about the meaning/value of the numbers that are written



# TAKING CUES FOR DESIGN POSSIBILITIES FROM SIMULATION ASSESSMENTS



# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: DURING THE SIMULATION



# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: DURING THE SIMULATION

The teacher in the video:

- used the interaction to ask questions about the student's approach and understanding
  - How did the 4 become the 14? Where did the ten come from?
  - How did you change the other number because that is something that I missed? (understanding the little 2)
  - Because I have never seen this approach before, why do you think you can change both of these numbers?
- was able to confirm or gain insight into the method used by the student
  - "I kept pushing because I didn't understand where this ten had come from"

# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: DURING THE SIMULATION

The teacher in the video:

- used the interaction to ask questions about the student's approach and understanding
  - How did the 4 b
  - How did you t missed?
  - Because I n change bo
- was able to confirm the student's understanding by the student
  - "I kept pushing because I didn't understand where this ten had come from"

The "student's" way of engaging in the situation impacted the learning opportunities of the teacher.

# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: DURING THE SIMULATION

How could the student's way of engaging in the simulation change to provide opportunities to develop MKT?

- Student volunteers an explanation of a process being used
- Student asks the preservice teacher a question about the process or to use a representation
- Student expresses frustration with an approach and wanting to know “how it works”
- Student wonders if the process works with other numbers



# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: DURING THE SIMULATION

Choose one of the options below and discuss how it would produce/effect an opportunity to learn MKT:

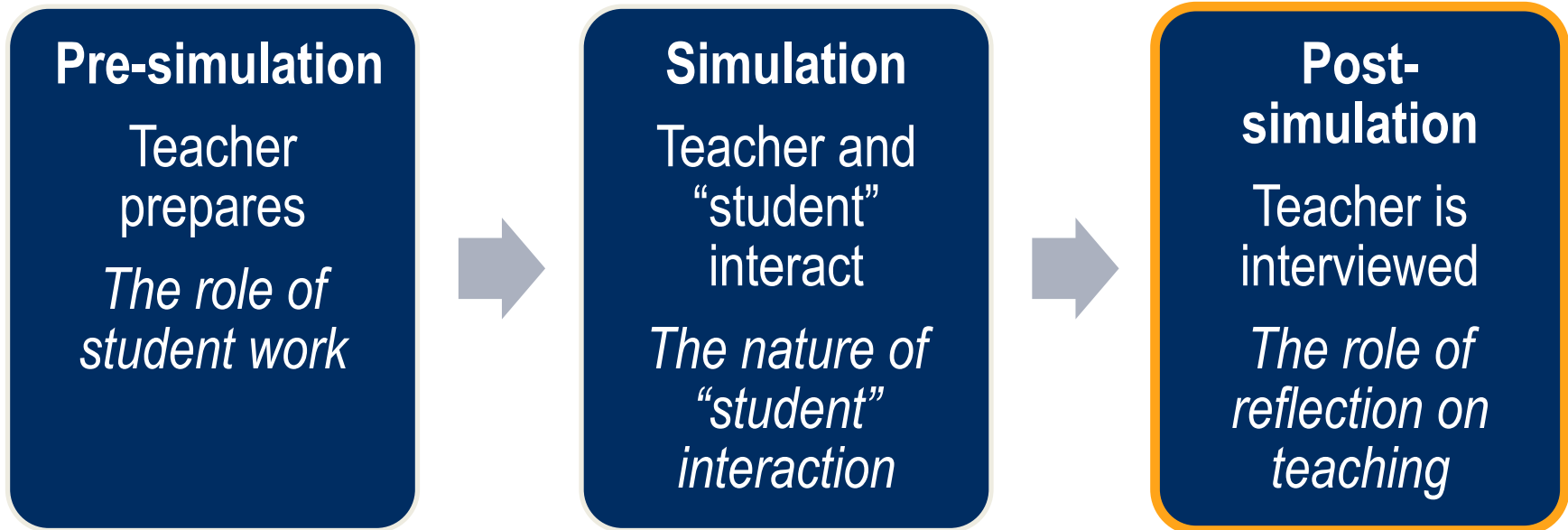
- Student volunteers an explanation of a process being used
- Student asks the preservice teacher a question about the process or to use a representation
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# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: DURING THE SIMULATION

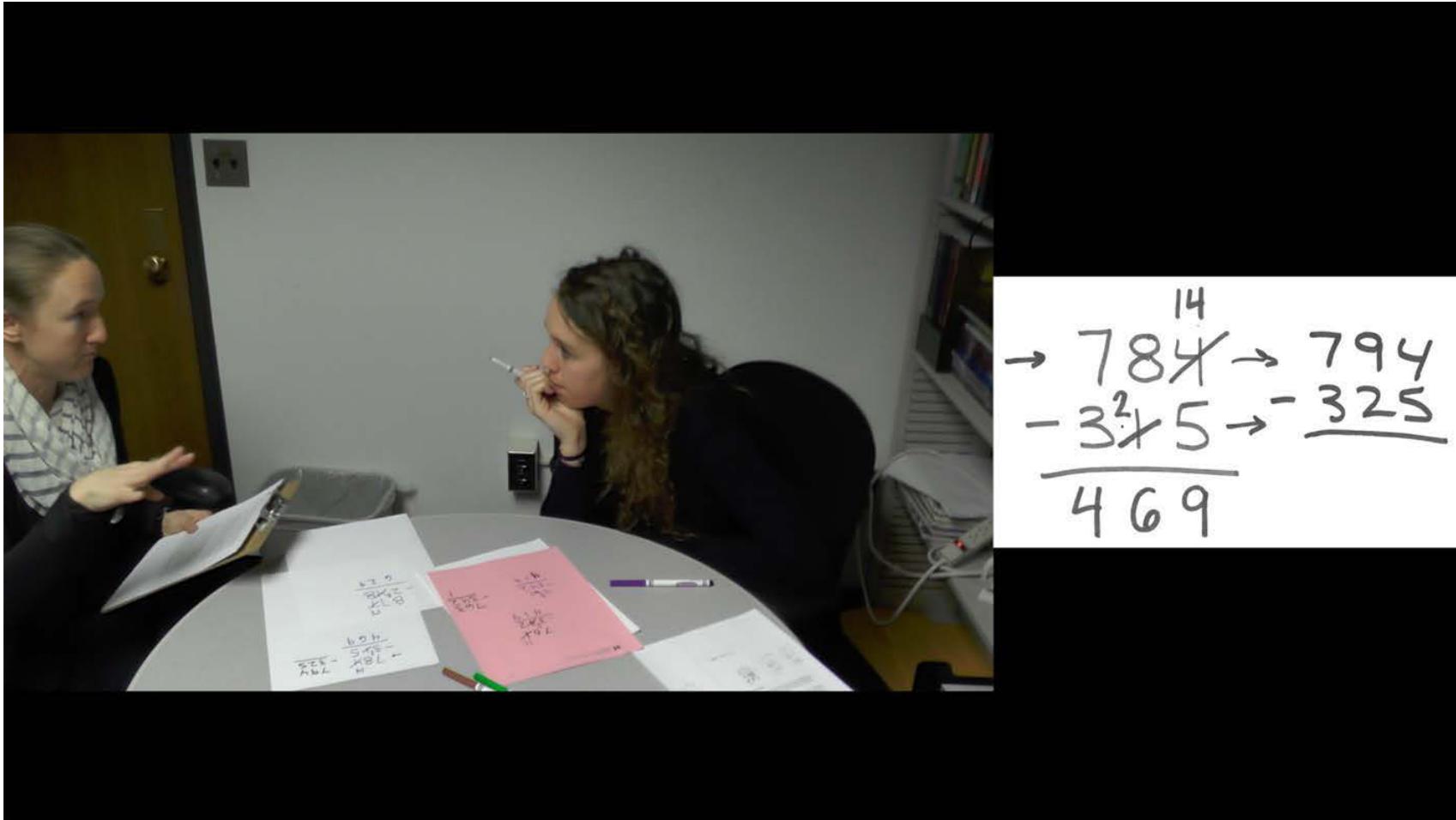
- Student volunteers an explanation of a process being used
  - Insures that information pertinent to understanding is available
- Student asks the preservice teacher a question about the process or to use a representation
  - Presses the preservice teacher to think through particular aspects of the process
- Student expresses frustration with an approach and wanting to know “how it works”
  - Entices the preservice teacher to provide an explanation
- Student wonders if the process works with other numbers
  - Presents the opportunity to generate numerical examples and/or to think about mathematical generalizations

# TAKING CUES FOR DESIGN POSSIBILITIES FROM SIMULATION ASSESSMENTS





# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: POST-SIMULATION



# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: POST-SIMULATION

There are many pieces beyond the interpreting interview that could be included after the simulation to enhance the preservice teacher's opportunities to develop MKT.

Which pieces do you feel would be most useful for supporting the development of MKT? What are some challenges of these approaches?

- Video replay (of the teacher or another teacher in the situation)
- Problem sets where the same process could be used and reflected upon
- Targeted resources (providing a reading)

# DESIGN OPPORTUNITIES TO SUPPORT LEARNING: POST-SIMULATION

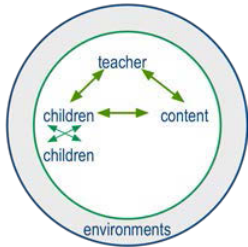
- Video replay
  - Reminding of the mathematical aspects of the interaction that could be reflected upon
- Video of another teacher in the situation
  - Illustrating additional mathematics aspects of the student's approach
- A problem set where the same process could be tried and reflected upon
  - Exploring how the process works in situations beyond those explored in the simulation
- Targeted resources (providing a reading on algorithms)
  - Considering a strategically chosen unpacking of the process and the way it works mathematically

# OUR NEXT STEPS

To study the potential of simulations as learning opportunities, we will be

- Enhancing the design of the simulations that had been used as assessments and pre/post simulation activities to enhance opportunities to develop MKT
- Developing/selecting instruments that will support understanding of changes to MKT, such as changes to:
  - knowledge of particular mathematical approaches,
  - knowledge of a pattern of student thinking, and/or
  - mathematical disposition

# CREDITS



Graphic on slide 2:

Cohen, D. K., Raudenbusch, S., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, 25 (2), 119-142.