HOW DO PRESERVICE TEACHERS ELICIT THE THINKING OF A STUDENT WHO HAS MADE A MISTAKE?

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We report on a study of novice teachers' eliciting performances in a scenario in which a student has made a mistake and, if sufficiently probed, is able to recognize the mistake and revise their work. Our findings reveal the skills and capabilities of one group of preservice teachers at the start of a teacher education program.

Keywords: Teacher Education-Preservice; Instructional Activities and Practices

In mathematics teaching, a regular problem of practice is that a student does something "unexpected" when solving a mathematics problem such as using an invented approach or making a "mistake" when carrying out a known process. Interactions around "unexpected" student responses can be powerful sites of learning for both teachers and students (Hiebert et al, 1997). For example, when an answer is incorrect, the joint sense-making required to interpret student thinking goes beyond the identification and correction of mistakes or errors into the conceptual analysis of why the mistake or error was made (Borasi, 1994; Kazemi & Stipek, 2001). The information that teachers can uncover through elicitation can then guide their pedagogical response. While the importance of interacting with students around mistakes is well-documented, far less is known about the ways in which preservice teachers (PSTs) elicit student thinking when a student has made a mistake in their arithmetic process. In this study, we set out to explore the eliciting skills and capabilities of PSTs in a context in which a student has made a mistake in their arithmetic process and will recognize the mistake if asked specific questions about their reasoning. We use the term *mistake* to mean isolated and unrepresentative mis-executions of an algorithm (i.e. "careless mistakes") (Radatz, 1980). We studied the skills of PSTs at the beginning of a teacher education program in order to find out about the skills that they bring to teacher education.

The Practice of Eliciting Student Thinking

In teaching, "teachers pose questions or tasks that provoke or allow students to share their thinking about specific academic content in order to evaluate student understanding, guide instructional decisions, and surface ideas that will benefit other students" (TeachingWorks, 2011). Our decomposition of the work of eliciting student thinking includes bringing out the student's process and probing key aspects of what the student says and does to uncover the student's understanding (Shaughnessy & Boerst, 2018b). We use "skill with eliciting student thinking" to refer to the degree to which PSTs are able to engage in these areas of work.

Using a Teaching Simulation to Assess Skills with Eliciting Student Thinking

Since 2011, we have been using teaching simulations to study PSTs' skill with eliciting student thinking (Shaughnessy & Boerst, 2018a Shaughnessy, Boerst, & Farmer, 2018). In these simulations, a PST interacts with a "standardized student" (a teacher educator taking on the role of a student using a well-defined set of rules for responding) around a specific piece of written work. We design teaching simulations to have a consistent three-part format (Shaughnessy & Boerst, 2018b). First, PSTs are provided with student work on a problem and given 10 minutes to prepare for an interaction. Second, PSTs have five minutes to interact with the standardized student, eliciting and probing the "student's" thinking to understand the steps they took, why they performed particular steps, and their understanding of the key mathematical ideas involved. Third, PSTs are interviewed about their interpretations of the student's thinking.

We designed this simulation to be one in which a student makes a mistake that is evident in their original written work. This student uses an alternative algorithm for solving multi-digit subtraction problems (see Figure 1). The process involves writing the value of the minuend and subtrahend in expanded form and making any necessary trades. When used correctly, the user would then subtract the numbers place-by-place in expanded form. This student correctly applies the expanding and trading process, but mistakenly adds values by place instead of subtracting. This student has conceptual understanding of expanded form, the meanings of addition and subtraction, and when, how, and why to make trades. In this particular instance, however, the student accidentally loses sight of the fact that they are solving a subtraction problem due to the addition symbols in expanded form. During the interaction, the student will realize their mistake when pressed to: make and evaluate an estimate for the original problem, represent their process with a picture, talk about the meaning of the operation, explain why trades were made, solve another multi-digit subtraction problem, or re-solve the original problem. The teacher educators carrying out the "student" role were trained to only reveal the mistake or revise the process if pressed by the PST on one or more of these specific points.

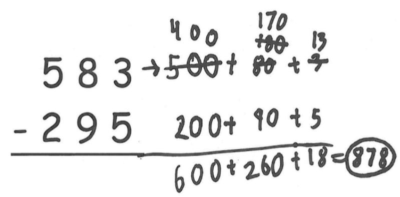

Figure 1. Student work. The student makes a mistake.

Methods

Thirty PSTs enrolled in a university-based teacher education program in the United States participated during the first week of the teacher education program. The teaching simulations were video-recorded. In this paper, we focus on three core components that we assessed: (a) eliciting the student's original process, (b) probing the student's understanding of key mathematical ideas, (c) eliciting and probing the student's mistake, including the reason for the mistake and the revised process and solution. For each of these components, we identified specific "moves" and tracked their presence or absence in each performance. We used the software package Studiocode© to parse the video into talk turns. Then, we identified "instances," which contain a question posed by a PST and the student's response to that question. Two independent coders applied all of the relevant codes to each instance. Disagreements were resolved through discussion and by referencing a code book.

Findings

Eliciting the Student's Original Process

The student's process had five steps: expanding both the minuend and subtrahend, comparing the numbers in each place, trading, adding (rather than subtracting) numbers by place, and adding the partial sums to arrive at the answer. The highest rates of eliciting occurred around the expansion (70%), the comparison of the numbers in each place (90%), and the trading steps (80%). In fact, 65% of PSTs elicited all three of these steps and 90% of the PSTs elicited two or more. However, only 53% of the PSTs elicited the step where the student made the mistake. The smallest percentage of eliciting occurred around the adding of the partial sums (10%). Because this step took place after the mistake, it may not have seemed important to PSTs who had elicited the mistake. Given that these PSTs elicited some, but not all, of the student's steps, we concluded that they brought skills relevant to eliciting a student's process that may be leveraged in the teacher education program to work towards more thorough elicitation.

Probing the Student's Understanding of the Original Process

We coded whether PSTs probed the student's understanding of six mathematical ideas underlying the process. The highest percentages of probing occurred around the operation in the problem (27%), why the student expanded (37%), and why the student traded (27%). The lowest percentages of probing occurred around the equivalence of expanded form and the "original" number (7%), the reasonableness of the answer (7%) and what trading means (0%). This suggests that limited probing of mathematical ideas occurred. However, when we looked across the set of ideas, we found that 67% of PSTs probed the student's understanding of one or more idea. Thus, we concluded that this 67% of PSTs brought skills relevant to probing the student's understanding in this scenario, but note that their probing was not focused on particular ideas.

Eliciting and Probing the Student's Mistake and Revised Process

We coded the extent to which the PSTs elicited and probed the student's mistake of adding numbers by place instead of subtracting. We found three patterns in PST' performances: eliciting that the student had made a mistake (47% of PSTs), pointing out the mistake and getting the student to agree without first eliciting the mistake from the student (20% of PSTs), and only asking questions that were not focused on the mistake (33% of PSTs). In other words, 67% of the PSTs uncovered the mistake either by eliciting it or asking the student to confirm that they made a mistake; 33% percent did not learn about the mistake.

We further examined the performances of the 47% of PSTs who elicited the mistake from the student. These 14 PSTs elicited the mistake in different ways. Eleven of the 14 asked about the operation involved in the problem and then pressed on how that operation (subtraction) related to what that the student had done (addition). Another two PSTs posed another problem for the student to solve or asked the student to redo the original problem. A third approach, used by one PST, was to ask why the student had made the trades (to get enough to subtract).

To explore what PSTs did after learning the student had made a mistake, we considered the 20 cases in which PSTs uncovered the student's mistake either by eliciting or pointing out the mistake. Three of the 20 PSTs asked questions to learn about how and/or why the mistake was made. For example, in response to the student's comment, "I think I made a mistake," one PSTs asked, "How did you make a mistake?" Five of the 20 PSTs made statements based on their own inferences about how and/or why the student made the mistake and got the student to agree. The other 12 PSTs did not ask follow-up questions about how or why the mistake was made. For example, one PST elicited the mistake, then immediately shifted focus to the revised process. Of the 20 PSTs, 15 elicited some or all of the revised process from the student. Sometimes this included asking about all of the steps in the process again, but other times it focused on redoing the step where the student had made the mistake. In 10 of the 20 cases, the student revised their final answer. These findings suggest that this group of PSTs bring skills relevant for eliciting a student's mistake but may need to learn reasons and strategies for probing how and why a mistake was made.

Discussion

Understanding the skills with teaching practices that novices bring to teacher education is key to designing experiences that are responsive to the needs of PSTs (Shaughnessy & Boerst, 2018a). This study examined the ways in which PSTs at the beginning of a teacher preparation program elicited the thinking of a student who made a mistake when using an alternative algorithm to solve a subtraction problem. Learning about the reason for a mistake is important for teachers to accurately assess and respond to student thinking. For example, a student who was genuinely unsure of the meaning of expanded form would need different instruction than a student who has made a mistake when carrying out a well-understood process. However, at the start of their teacher education program, these PSTs focused more on eliciting the revised method and/or solution than asking about why the mistake was made. The data suggest several directions for continuing research. First, the PSTs in this study were in their first week of a teacher education program. In what ways do their skills with eliciting student thinking develop over time? Second, what course activities might effectively cultivate an inclination to probe how and why students make mistakes? Third, the assessment itself involved a mistake where the student was using a "non-standard" approach to solve the problem. Anecdotally, we have reason to think that some PSTs were discounting the student's reasoning because they believed that the student should be using a different method to solve the subtraction problem. A future study could compare skills in eliciting around a mistake with a "standard" and an "alternative" algorithm.

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