ASSESSING MATHEMATICAL KNOWLEDGE FOR TEACHING IN SIMULATIONS

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Association of Mathematics Teacher Educators Annual Meeting
February 12, 2021

The research reported here was supported by the National Science Foundation, through a grant to the University of Michigan. The opinions, findings, and recommendations expressed are those of the authors and do not represent views of the National Science Foundation.

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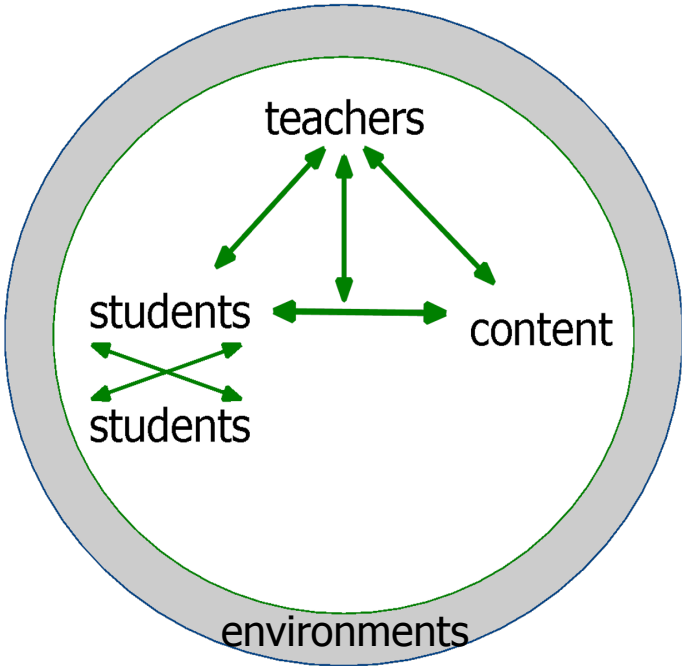
TEACHING AS INTERACTION



Cohen, Raudenbush, & Ball, 2003
Lampert, 2001

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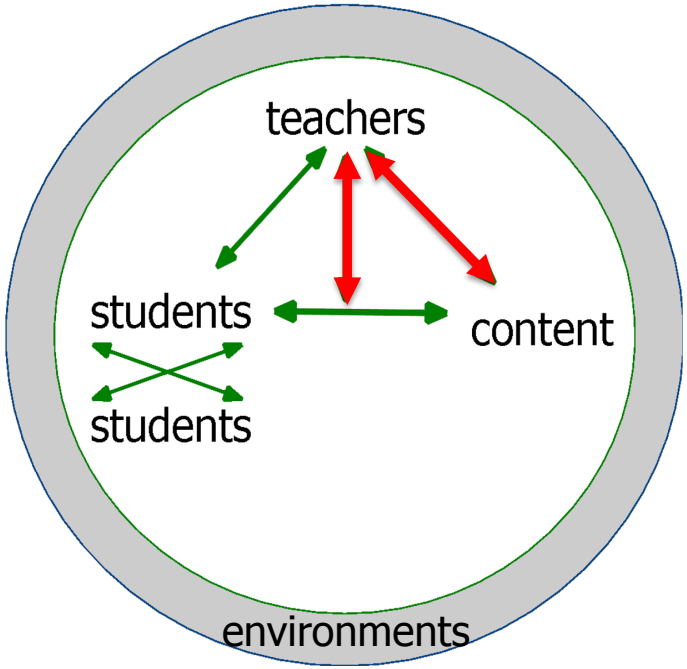

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MATHEMATICAL KNOWLEDGE FOR TEACHING

Mathematical knowledge as it is used in the work of teaching is crucial for many tasks, including:

- Choosing examples
- Developing follow-up problems
- Explaining ideas to students and supporting explanations
- Using representations and materials to show the meaning of mathematical ideas
- Posing questions
- Interpreting students' thinking and choosing what to say or do in response

ELICITING STUDENT THINKING

A core teaching practice: to find out what students know or understand, and how they are thinking/reasoning

- Establishing an environment in which a student is comfortable sharing his/her thinking
- Posing questions to get students to talk
- Listening to and hearing what students say
- Probing students' responses
- Developing an idea of what a student thinks
- Checking one's interpretation

ASSESSING MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

Many methods can and have been used to assess MKT, including:

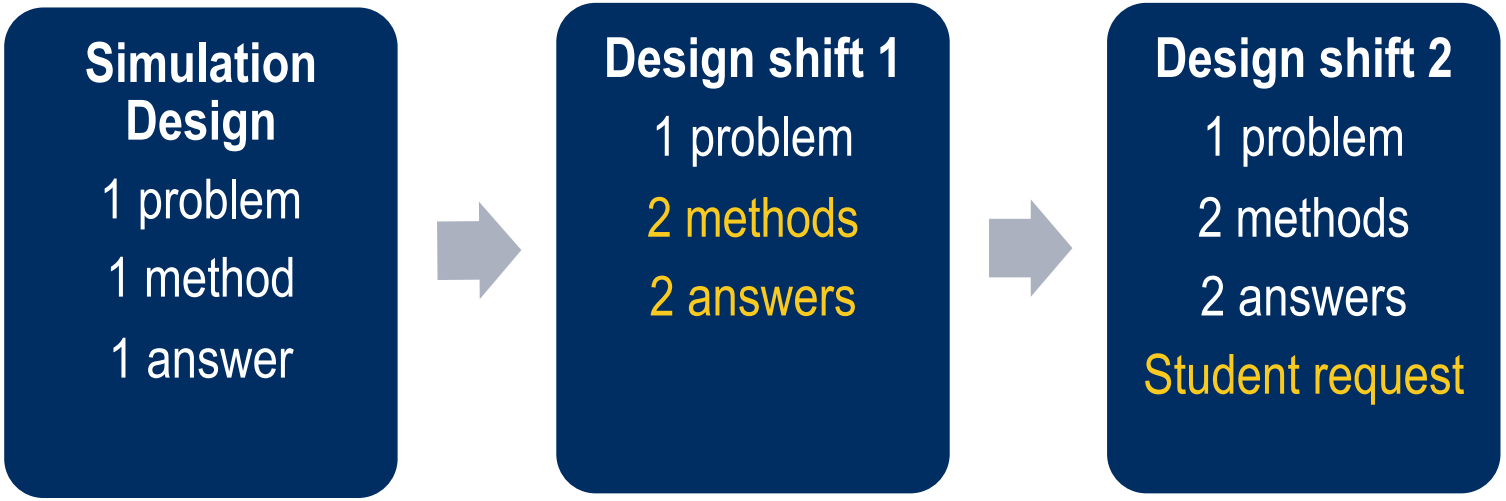
- Responding to multiple choice surveys
- Analyzing of video of students solving mathematics problems
- Critiquing lessons, units, and trajectories in mathematics curriculum materials
- Creating of mathematics problems for students to solve
- Rehearsing the use of manipulatives to explain a mathematical process

USING SIMULATIONS TO ASSESS MKT *IN INTERACTION*

Simulations:

- are approximations of practice
- are commonly used in many professional fields
- place authentic, practice-based demands on a participant
- purposefully suspend or standardize some elements of the practice-based situation
- can provide preservice teachers with insights that are not possible or practical to generate in real-life professional contexts

SESSION OVERVIEW



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A TEACHING SIMULATION DESIGN

Simulation Design
1 problem
1 method
1 answer

Design shift 1
1 problem
2 methods
2 answers

Design shift 2
1 problem
2 methods
2 answers
Student request

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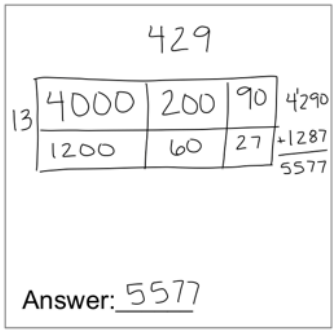
THE PRESERVICE TEACHER PREPARES FOR A SIMULATION

The preservice teacher:

1. **prepares for an interaction with a standardized student focused on a work sample**



Your goal is to elicit and probe to find out what the "student" did to produce the answers as well as the way in which the student understands the steps that were performed.

Correct answer, alternative algorithm, degree of understanding is unclear

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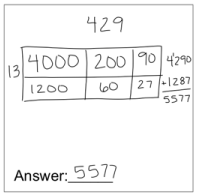
THE PRESERVICE TEACHER ENGAGES IN A SIMULATION



The preservice teacher:

1. prepares for an interaction with a standardized student about one piece of student work
2. **interacts with the student to probes the standardized student's thinking**




A Standardized Student

Developed response guidelines focused on:

- what the student is thinking such as
  - using a diagram to organize the multiplication of two multidigit numbers (the approach looks like an area model)
  - appling the method correctly with conceptual understanding of the procedure
- general orientations towards responses such as
  - knowing multiplication facts and does not make basic facts errors.
  - giving the least amount of information that is still responsive to the question
- responses to anticipated questions

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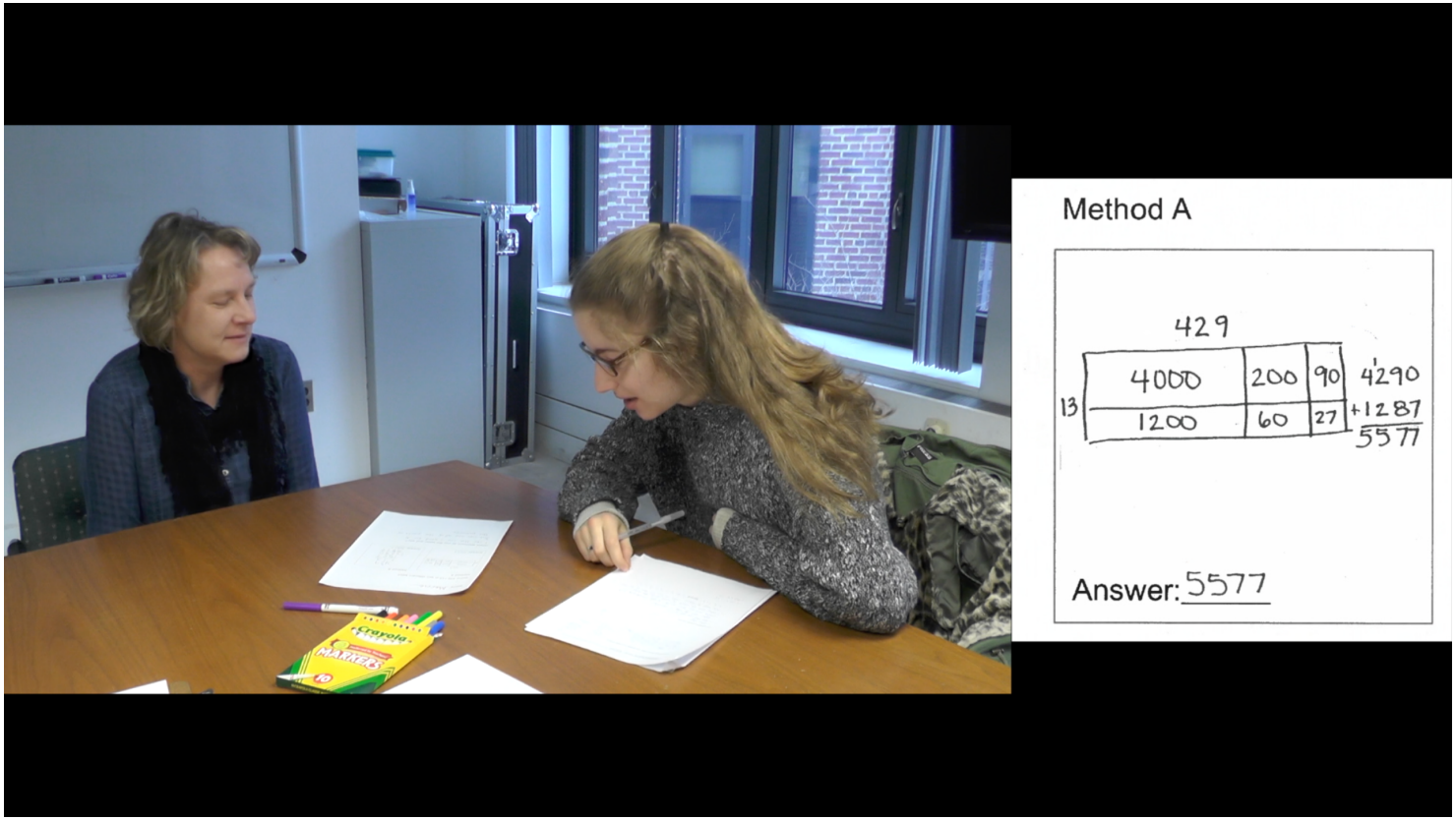
WHAT OPPORTUNITIES EXIST TO ASSESS THE PRESERVICE TEACHER'S MKT?



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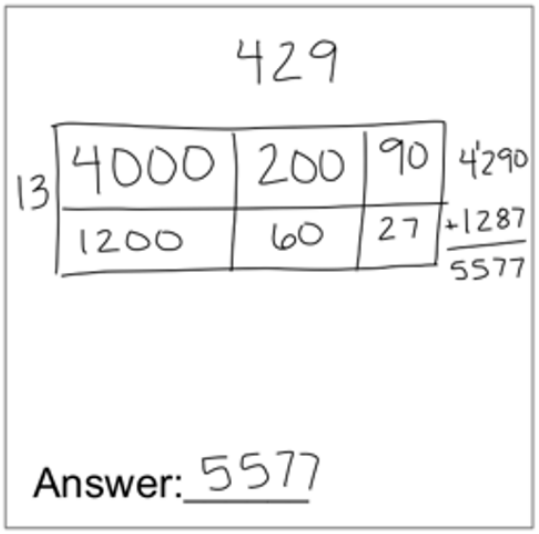
WHAT OPPORTUNITIES EXIST TO ASSESS THE PRESERVICE TEACHER'S MKT?

Add your ideas to the chat window



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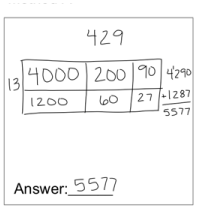

The preservice teacher:

- Elicits information about how the diagram maps onto the numbers from the given problem
- Probes mathematics that is crucial for understanding the method
  - Why did you multiply 3 by these numbers?
  - Do you know why you added those numbers?

SHIFTING THE SIMULATION DESIGN

Simulation Design

1 problem

1 method

1 answer

Design shift 1

1 problem

2 methods

2 answers

Design shift 2

1 problem

2 methods

2 answers

Student request

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DESIGN CHANGE #1

What might having a student's work on two problems afford for making their MKT visible?

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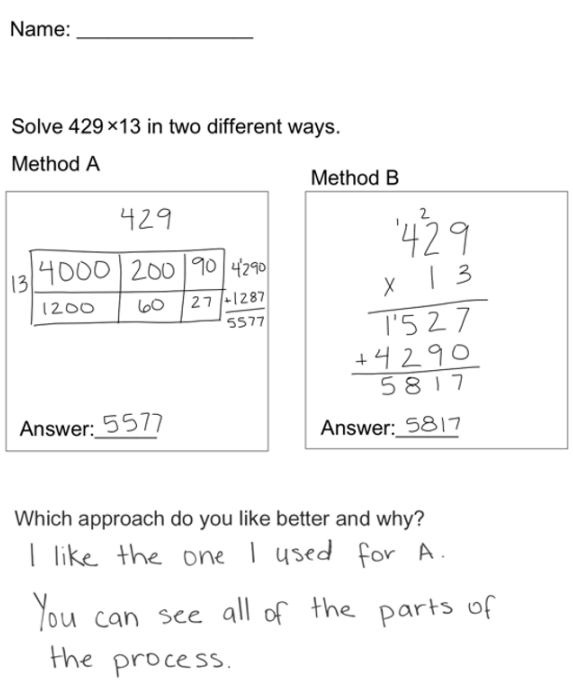

Name: _____

Solve 429×13 in two different ways.

Method A

429

13	4000	200	90	4290
	1200	60	27	+1287
				5577

Answer: 5577

Method B

$$\begin{array}{r} {}^{1}{}^{2}429 \\ \times\ 13 \\ \hline 1527 \\ +4290 \\ \hline 5817 \end{array}$$

Answer: 5817

Which approach do you like better and why?

I like the one I used for A.

You can see all of the parts of the process.

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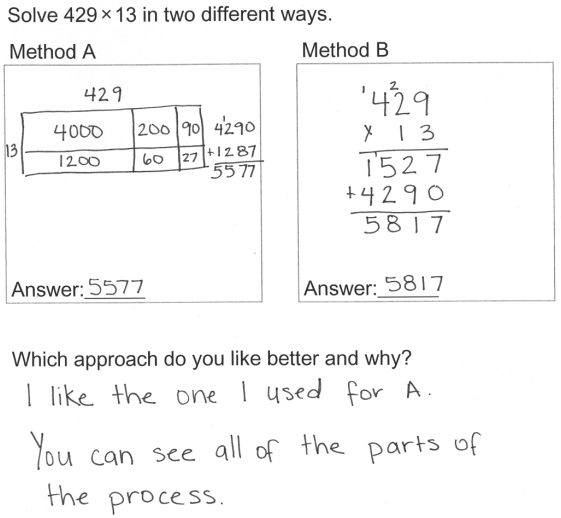
THE PRESERVICE TEACHER PREPARES FOR A SIMULATION

The preservice teacher:

1. **prepares for an interaction with a standardized student about two pieces of student work**



Your goal is to elicit and probe to find out what the "student" did to produce the answers as well as the way in which the student understands the steps that were performed.

- **Method A: Correct answer, alternative algorithm, degree of understanding is unclear**
- **Method B: Incorrect answer, "standard" algorithm, degree of understanding is unclear**

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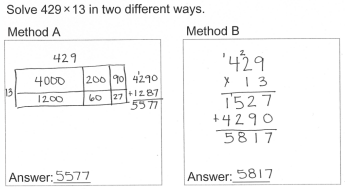
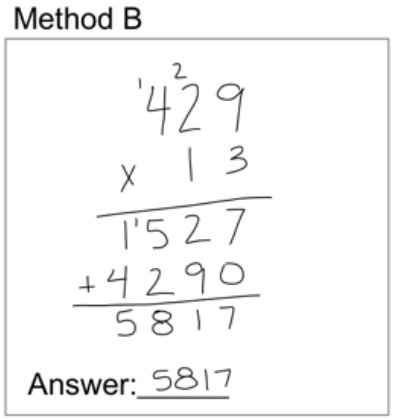
THE PRESERVICE TEACHER ENGAGES IN A SIMULATION



The preservice teacher:

1. prepares for an interaction with a standardized student about two piece of student work
2. **interacts with the student to probes the standardized student's thinking**



A Standardized Student

Developed response guidelines focused on:

- what the student is thinking including
  - their process and understanding of method A and method B
  - that they prefer Approach A because "you can see all of the parts of the process"
- general orientations towards responses
  - give the least amount of information possible that is still responsive to the question asked by the preservice teacher
- responses to anticipated questions

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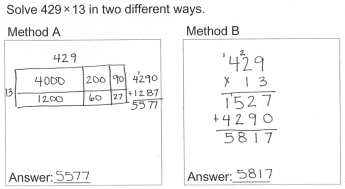
THE PRESERVICE TEACHER ENGAGES IN A SIMULATION

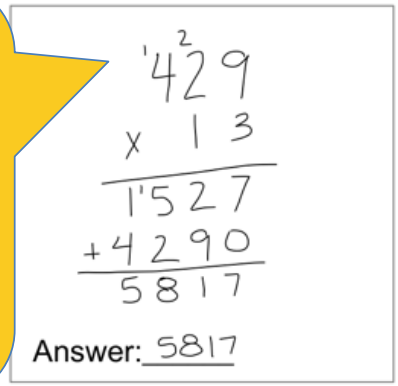


The preservice teacher:

1. prepares for an interaction with standardized student about two of student work
2. **interacts with the student to probes the standardized stud thinking**

- Uses the "traditional" approach for multiplication, but adds any carry before multiplying (instead of multiplying before adding);
- Has conceptual understanding of parts of the procedure (e.g., why you need to carry)
- Believes you can add or multiply in either order

Method B



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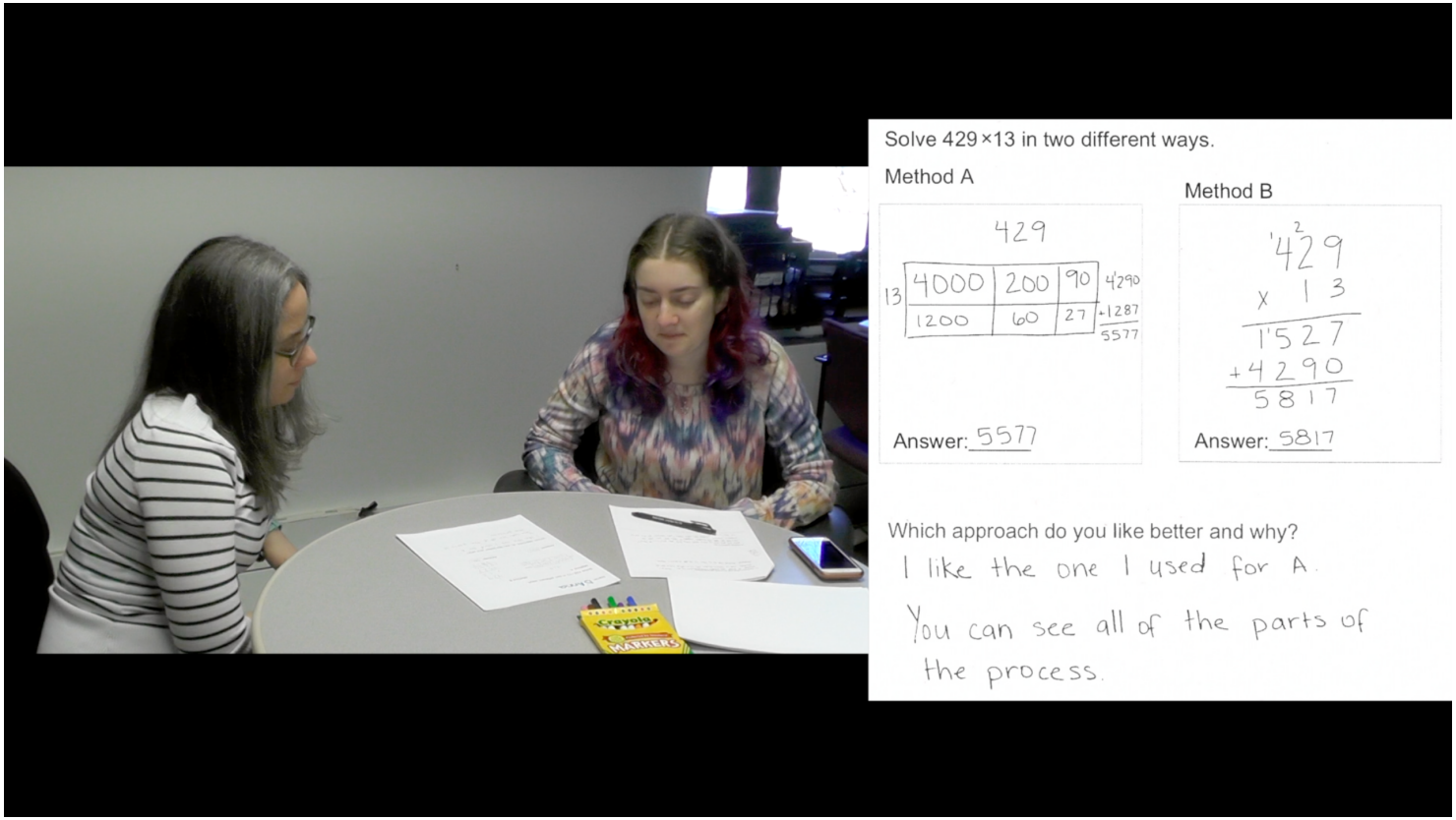

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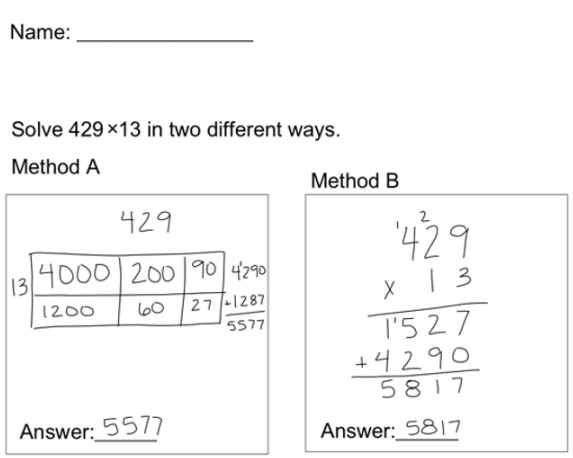
Add your ideas to the chat window



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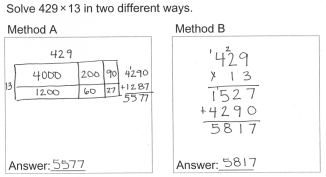

The preservice teacher:

- Probes mathematics that is crucial for understanding both methods
  - Why did you multiply 3 by this number? [Method A]
  - Do you know why we added these two numbers together? [Method B]
- Probes the student's understanding about why the carry should be added
  - Why did you add the 2 and this one together?

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SHIFTING THE SIMULATION DESIGN AGAIN

Simulation Design

1 problem

1 method

1 answer

Design shift 1

1 problem

2 methods

2 answers

Design shift 2

1 problem

2 methods

2 answers

Student request

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DESIGN CHANGE #2

What might having the student request help in understanding why the approaches resulted in different answers afford for seeing their MKT in use?

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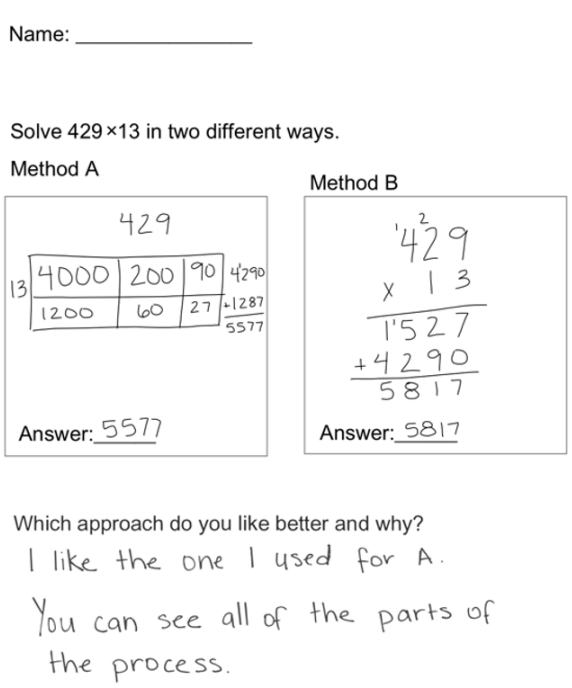

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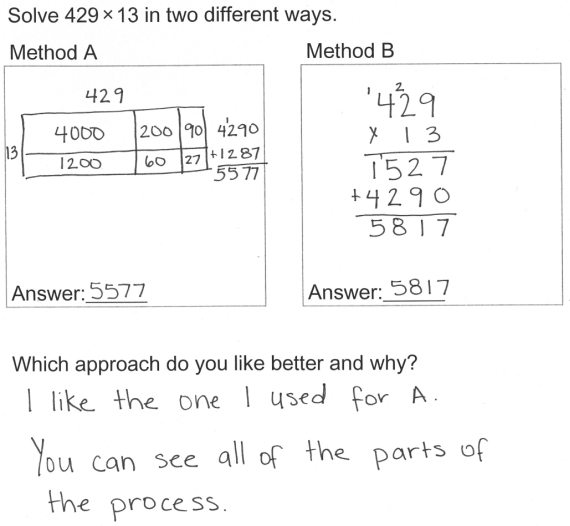

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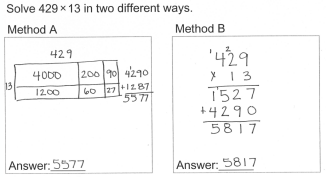

The preservice teacher:

1. prepares for an interaction with a standardized student about one piece of student work
2. **interacts with the student to probes the standardized student's thinking**

A Standardized Student

Adjusted the general orientations towards responses guidelines :

- Actively conveying that it is a problem that the two approaches generated different answers; demonstrating that sometimes they result in the same answer
- Actively displaying a "positive disposition" towards mathematics by articulating that multiple approaches for working on problems is useful and that they are sure that they can understand how to use method B
- Actively requesting that the preservice teacher help them understand what went wrong with method B
- Actively explaining why understanding is important
- Actively pressing the preservice teacher on "why" if they are told to follow a rule

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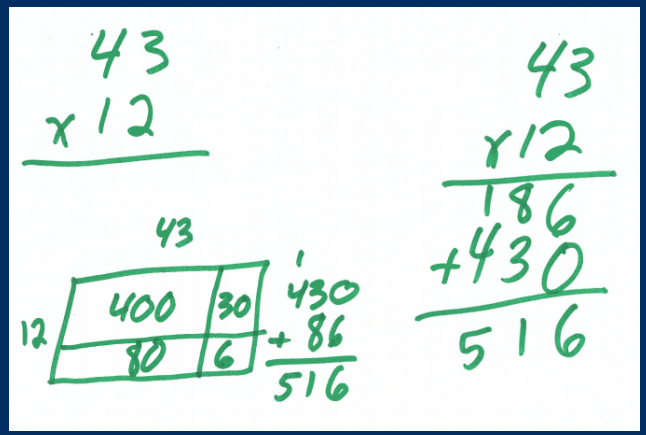
THE STUDENT FRAMES THAT "SOMETHING IS WRONG" WITH THEIR PROCESS

After the steps of the traditional process are shared, the student says:

> This answer here is different from when I used the other way. I don't think this is right, but I know this way works. Like if I multiply 43 x 12.

The student proceeds to show the preservice teacher that they get the same answer using both approaches for 43 x 12. The student talks aloud as they work.



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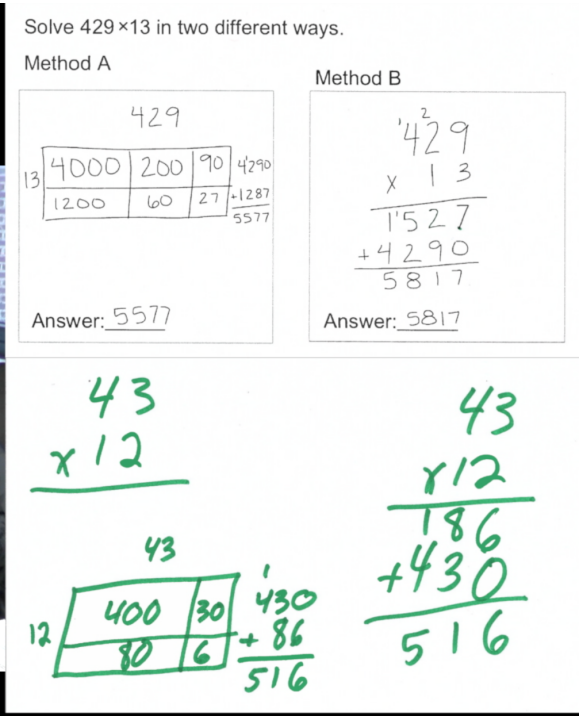

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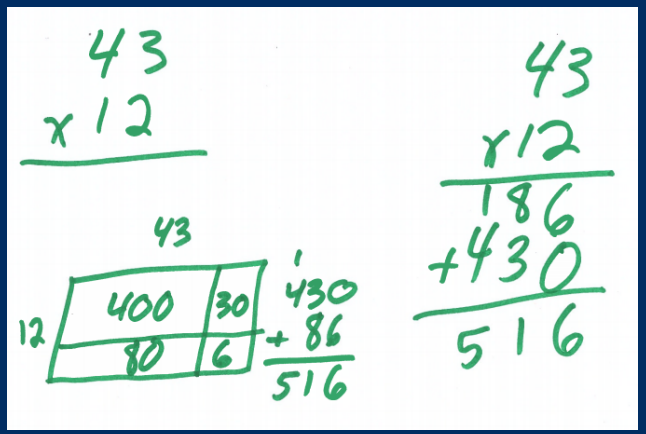
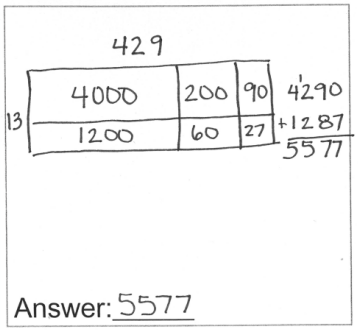
"in the moment" problem

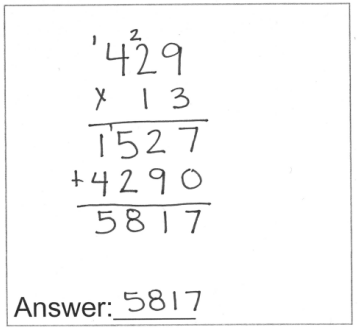


Solve 429 × 13 in two different ways.

Method A



Method B



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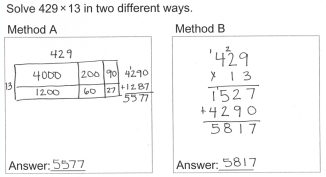

The preservice teacher:

- Probes mathematics that is crucial for understanding one/both methods (before the clip starts)
- Probes connections between problems as well as methods
  - How are these two problems different? What did you do over here [original work] that you didn't do over here [in the moment problem]
  - Where are the parts of method A in method B? Can you show me where the 4,290 and 1,287 are?
- Response to the student's request for an explanation of why they should "multiply before adding the carry"
  - I just memorized that rule….You should get a math teacher

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IMPLICATIONS AND NEXT STEPS

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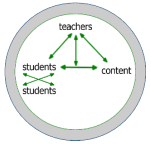
IMPLICATIONS AND NEXT STEPS

- When designing teaching simulations, features can be adjusted to make MKT more visible, such as:
  - The number of problems that the student has completed
  - The student role, shifting from responding to the teacher's questions to asking questions of the teacher
- Noticing the ways in which the preservice teachers engaged in the enhanced simulations suggests:
  - further research on how PSTs leverage multiple examples of student work to support questioning/explaining
  - work to design and use TE experiences in which preservice teachers practice generating intellectually honest explanations of mathematical concepts that are connected with common student patterns of thinking

CREDITS



Cohen, D. K., Raudenbusch, S., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25 (2)*, 119-142.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

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